Infra Soft Topological Spaces: explanation of New Soft Structure

Dr.Vijay Kumar
Associate Professor,
Department of Mathematics,
P.G Centre, College of Commerce,Arts and Science, Patna

Abstract

Finding the weakest conditions that retain some topologically inspired traits is always convenient. In order to do this, we introduce the idea of an infra soft topology, which is a group of subsets that extends the notion of soft topology by doing away with the presumption that the group is closed under arbitrary unions. Our research focuses on the fundamental ideas of infra soft topological spaces, including infra soft open and closed sets, infra soft interior and closure operators, and infra soft limit and boundary points of a soft set. With the aid of some clarifying examples, we demonstrate the key characteristics of these ideas. We next go over a few techniques for creating infra soft topologies, including infra soft neighbourhood systems, infra soft topology's fundamentals, and infra soft relative topology. We also look into how to start a crisp infrared topology into an infrared soft topology. Finally, we examine the idea of continuity between infra soft topological spaces and establish the circumstances under which the continuity between an infra soft topological space and its parametric infra topological spaces is preserved.

Keywords: Topological Spaces, Infra Soft, Soft Structure, soft set theory.

Introduction

Infra topology and soft set theory, two fields that intersect in this study. Our contribution's framework is infra soft topology, an intriguing structure that was created as a result of their hybridization. Let's summarise the topic's history and current state of knowledge**.**

In 1999, Molodtsov developed the idea of soft sets as a novel mathematical strategy to deal with problems involving uncertainties, and he described how soft sets may be used to solve a wide range of issues in several fields. Due to its wide range of applications, this theory has drawn considerable interest from scientists and researchers. In the past few years, the field of soft sets research has grown rapidly; for instance, Maji et al. proposed a number of soft operations in 2003, including union, intersection, subset, and equality relations between two soft sets. As a soft variation of the empty and universal crisp sets, they also established the null and absolute soft sets. Ali et al. introduced some new operations on soft sets, studied their primary features, and demonstrated the shortcomings mentioned in. New sorts of soft equality were described by Abbas et al. and Qin and Hong, who used them to create new varieties of algebraic structures. New kinds of operations between soft sets have recently been developed and addressed by Al-Shami and El-Shafei.

2011 saw the introduction of a topological structure on soft setting by Shabir and Naz. The basic concepts of soft topologies, including as soft open and closed sets, soft subspaces, and belonging and nonbelonging relations, were defined. These concepts are used to launch soft separation axioms. The concept of a soft point was developed by Zorlutuna et al. and is useful for studying various characteristics of soft interior points and soft neighbourhood systems. Samanta et al. independently reformulated this idea, with Nazmul and Samanta using it to discuss soft neighbourhood systems and uncover some relationships between soft limit points of a soft set, while Das and Samanta used the new version of the soft point to study the idea of soft metric spaces. Many academics have examined the characteristics of soft topologies and contrasted their effectiveness with that of classical topologies; for instance, generalisations of open sets have been studied in soft topologies. We disproved various claims made in relation to the soft separation axioms, particularly those defined by soft points. Weakening a soft topology's criteria resulted in some generalisations of a soft topology. By ignoring a soft topology's finite intersection constraint, El-Sheikh and Abd El-Latif, for instance, introduced the concept of super soft topological spaces in 2014. As a result, this route drew many researchers who investigated key ideas related to supra soft topologies. For instance, Thomas and John developed the idea of soft generalised topological spaces, which is defined as a family of soft sets that satisfy an arbitrary union condition of a soft topology, and Zakari et al. developed the idea of soft weak structures, which is defined as a family of soft sets that contains the null soft set. When two soft topologies are identical, a concept called soft bitopological space—which Ittanagi studied—can be thought of as a soft topological space. As a recent expansion of soft topology, Al-Shami et al. created soft topology on ordered settings. Similar research was conducted on ordered settings by Al-Shami and El-Shafei on supra soft topology.

Infra soft topological spaces make it easier to initiate cases that illustrate linkages between certain topological concepts, and many aspects of soft topological spaces are still applicable on these spaces. We therefore intend to conduct an exhaustive investigation of infra soft topological spaces in this paper.

Infra Soft Topological Spaces

In this section, the idea of infra soft topology as a class of soft sets—which is closed under finite soft intersection and contains the null soft set—is introduced. It is independent of a supra soft topology and sits between soft topology and soft weak structure. We outline the key terms and characteristics of infra soft topology. One advantage of infra soft topology is the continued applicability of many soft topology results, particularly those that concern the soft interior and closure operators. Several examples are given to validate the results that were found.

Definition: The collection ϑ of soft sets over X under a fixed set of parameters E is said to be an infra soft topology on X if it is closed under finite soft intersection and the null soft set is a member of ϑ. The triple X, ϑ, E is called an infra soft topological space. Every member of ϑ is called an infra soft open set, and its relative complement is called an infra soft closed set.

The following examples elucidate the uniqueness of infra soft topology than the other celebrated soft structures.

Let $E = \{e_1, e_2\}$ of parameters and X ={a b} be the universal set. Then $\vartheta_a = \{\tilde{X}, F_E \sqsubseteq \tilde{X} : a \notin F_E\}$ is an infra soft topology on X; we called ϑ_a an excluding point infra soft topology. On the contrary, and VE � are and

$$U_E = \{(e_1, \{a\}), (e_2, \varnothing)\} \qquad V_E = \{(e_1, \varnothing), (e_2, \{a\})\}$$

infra soft open sets, but their union is not an infra soft open set. Therefore, ϑ_a is neither supra soft topology nor generalized soft topology. Hence, it is not a soft topology.

Let, $E = \{e_1, e_2\}$ f parameters and X ={a b} be the universal set. Then $U_E = \{(e_1, \{b\}), (e_2, X)\}$ is a supra soft topology on X; we called ϑ_a a particular point supra soft topology. On the contrary, and

$U_E = \{(e_1, \{b\}), (e_2, X)\}$ $V_E = \{(e_1, \tilde{X}), (e_2, \{b\})\}$

are supra soft open sets, but their intersection is not a supra soft open set.)erefore, ϑa is not an infra soft topology. Moreover, it is not a soft topology.

Proposition: Let fφ: S(XE) ⟶ S(YE) be a soft map. If ϑ is an infra soft topology on Y, then a class is an in$\theta = \{f_\varphi^{-1}(G_E) \sqsubseteq \tilde{X}: G_E \in \vartheta\}$

Proof: Since Φ and Y ∈ ϑ, then $f_\varphi^{-1}(\Phi) = \Phi$ ıd $f_\varphi^{-1}(\tilde{Y}) = \tilde{X} \in \theta$. E and $F_{2E} \in \theta$.Then, there exist H_{1E} and $H_{2E} \in \vartheta$ such that $f^{-1}{}_\varphi(H_{1E}) = F_{1E}$ and $f^{-1}{}_\varphi(H_{2E}) = F_{2E}$. Therefore, $F_{1E} \sqcap F_{2E} = f^{-1}{}_\varphi(H_{1E}) \sqcap f^{-1}{}_\varphi(H_{2E}) = f^{-1}{}_\varphi(H_{1E} \sqcap H_{2E})$. Since $H_{1E} \sqcap H_{2E} \in \vartheta$, then $F_{1E} \sqcap F_{2E} \in \theta$. Hence, the proof is complete.

Proposition: Let H_E be a soft subset of (X, ϑ, E). Then, the following properties hold.

(i) $P_e^x \in Int(H_E)$ if and only if there exists an infra soft open set G_E such that $P_e^x \in G_E \sqsubseteq H_E$

(ii) $P_e^x \in Cl(H_E)$ if $H_E \sqcap G_E \neq \Phi$ for every infra soft open set G_E containing P_e^x

Proof:

(i) Straightforward.

(ii) Necessity: $P_e^x \in \mathrm{Cl}(H_E)$. Then P_e^x belongs to every infra soft closed set containing H_E. Suppose that there exists an infra soft open set G_E containing P_e^xsuch that $H_E \sqcap G_E = \Phi$, so $H_E \sqsubseteq G_E^c$. This is a contradiction. Thus, the necessary part hold.

Sufficiency: let $P_c^x \notin \mathrm{Cl}(H_F)$ Then $(\mathrm{Cl}(H_E))^c$ is an infra soft open set containing P_e^x such that $H_E \sqcap (\mathrm{Cl}(H_E))^c = \Phi$. Thus, the proof is complete.

Proposition: Let H_E be a soft subset of (X, ϑ, E). Then,

(i) $(\mathrm{Int}(H_E))^c = Cl(H_E^c)$

(ii) $(\mathrm{Cl}(H_E))^c = \mathrm{Int}$

Proof:

(i) $(\mathrm{Int}(H_E))^c = \{\sqcup_{i\in I}(G_{iE}): G_{iE}$ E is an infra soft open set included in $H_E\}^c = \sqcap_{i\in I}\{G_E^c: G_E^c$

is an infra soft closed set including $H_E^c\} = \mathrm{Cl}(H_E^c)$

In a similar manner, we prove (ii).

Continuity between Infra Soft Topological Spaces

In this section, we define the concept of continuity between infra soft topological spaces and then give its equivalent conditions using infra soft open and infra soft closed sets. Also, we discuss losing some equivalent conditions of soft continuity on infra soft topology with the help of an ilillustrative example. We close this section by studying "transmission" of continuity between an infra soft topological space and its parametric infra topological spaces.

Definition: soft map f_φ from (X, ϑ, E) to (Y, μ, E) is said to be infra soft continuous at a soft point if for each i $P_e^x \in \tilde{X}$ ft open set U_E containing there $f_\varphi(P_e^x)$, fra soft open set V_E containing such that $f_\varphi(V_E) \sqsubseteq U_E$. P_e^x

A soft map f_φ: (X, ϑ, E) ⟶ (Y, μ, E) is infra soft continuous if and only if the inverse image of each infra soft open set is an infra soft open set.

Proof: Necessity: let U_E be an infra soft open subset of (Y, μ, E). Without loss of generality, consider $f^{-1}{}_\varphi(U_E) \neq \Phi$.) Then, for each $P^x{}_e \in f^{-1}{}_\varphi(U_E)$, we have an infra soft open subset V_E of (X, ϑ, E) containing such that P_e^x $f_\varphi(V_E) \sqsubseteq U_E$.

Thus, $P_e^x \in V_E \subseteq f_\varphi^{-1}(U_E)$ and $\sqcup\{V_E\} = f_\varphi^{-1}(U_E)$. Hence, $f^{-1}{}_\varphi(U_E)$ is infra soft open.

Sufficiency: suppose that $P_e^x \equiv$ X and U_E is an infra soft open set containing $f_\varphi($ $)$ P_e^x. Then, $f^{-1}{}_\varphi(U_E)$ is an infra soft open set containing such that $f_\varphi(f^{-1}{}_\varphi(P_e^x)) \sqsubseteq U_E$. Therefore, f_φ is infra soft continuous at which we choose arbitrarily; hence P_e^x, is infra soft continuous.

Conclusion

The idea of an infra soft topology has been introduced in this study as a new structure that is weaker than a soft topology. The main objective of researching this idea is to maintain some soft topological qualities with less circumstances than topology. In three different ways, we have helped to advance understanding in this field. In the beginning, we defined the fundamental ideas of infra soft topological spaces and examined properties. We have observed that whereas most interior and closure operator properties are invalid on other generalisations of soft topology, such as supra soft topology, they are generally valid on infra soft topological spaces. Second, we have suggested several methods for creating infra soft topologies, including crisp infra topol ogies, infra soft basis, infra soft subspace, and soft maps and soft neighbourhood systems. Since the features of soft topology and infra soft topology are equivalent, the techniques of soft neighbourhood systems and soft operators actually introduce soft topology.We have discussed the idea of continuity between infra soft topological spaces and conducted research on it. Using infra soft open and infra soft closed sets, we have discussed this idea. Additionally, we have demonstrated that several characterizations of continuity in soft topology, particularly those that are based on the interior and closure operators, are reliant on the frame of infra soft topology. . In further studies, we intend to establish soft topological notions including compactness, connectedness, and separation axioms on the foundation of infra soft topology. We specifically shed light on determining whether of these qualities still hold true on infra soft topologies.

References:

[1] D. Molodtsov, “Soft set theory-First results,” Computers & Mathematics with Applications, vol. 37, no. 4-5, pp. 19–31, 1999.

[2] N. Caˇgman and S. Enginoˇglu, “Soft matrix theory and its decision making,” Computers and Mathematics with Applications, vol. 59, pp. 3308–3314, 2010.

[3] S. Yuksel, T. Dizman, G. Yildizdan, and U. Sert, “Application of soft sets to diagnose the prostate cancer risk,” Journal of Inequalities and Applications, vol. 2013, p. 229, 2013.

[4] P. K. Maji, R. Biswas, and A. R. Roy, “Soft set theory,” Computers & Mathematics with Applications, vol. 45, no. 4-5, pp. 555–562, 2003

[5] M. I. Ali, F. Feng, X. Liu, W. K. Min, and M. Shabir, “On some new operations in soft set theory,” Computers & Mathematics with Applications, vol. 57, no. 9, pp. 1547–1553, 2009. [6] M. Abbas, M. Ali, and S. Romaguera, “Generalized operations in soft set theory via relaxed conditions on parameters,” Filomat, vol. 31, no. 19, pp. 5955–5964, 2017.

[7] K. Qin and Z. Hong, “On soft equality,” Journal of Computational and Applied Mathematics, vol. 234, no. 5, pp. 1347– 1355, 2010.

[8] T. M. Al-shami and M. E. El-Shafei, “T-soft equality relation,” Turkish Journal of Mathematics, vol. 44, no. 4, pp. 1427–1441, 2020.

[9] M. Shabir and M. Naz, “On soft topological spaces,” Computers & Mathematics with Applications, vol. 61, no. 7, pp. 1786–1799, 2011.

[10] I. Zorlutuna, M. Akdag, W. K. Min, and S. Atmaca, “Remarks on soft topological spaces,” Annals of Fuzzy Mathematics and Informatics, vol. 2, pp. 171–185, 2012.

[11] S. Das and S. K. Samanta, “Soft metric,” Annals of Fuzzy Mathematics and Informatics, vol. 6, no. 1, pp. 77–94, 2013.

[12] S. Nazmul and S. K. Samanta, “Neighbourhood properties of soft topological spaces,” Annals of Fuzzy Mathematics and Informatics, vol. 6, no. 1, pp. 1–15, 2013.