JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

# Study of C-Set in a Topological Vector Space

By
**Dr. Kumar Swetank**
Assistant Professor
P.G. Department of Mathematics
Patliputra University, Patna

## Abstract:-

The abstract for C-Set in a topological vector space if $\tau$ is a topology on a vector space X such that every point of X is a closed set and the vector space operations are continuous with respect to $\tau$. The $\tau$ is said to be a vector topology on X, and X is called a topological vector space. This condition also implies that $\tau$ is Hausdroff topology. The closure of $\bar{E}$ of $E \subset X$ is the intersection of all closed sets that contain E. The interior $E^0$ of E is the union of all open sets that are subset of E. Now we will study some theorems.

**Theorem-1:** Let X be a topological vector space. Let $A \subset X$ and A be C-Set. Then $\bar{A}$ is also C-Set.

**Proof:** First we show that if $\alpha$ is a scalar, then $\alpha\overline{\gamma} = \overline{\alpha\gamma}$ where $\gamma \subset X$ .

If $\alpha = 0$ then two sets are obviously equal, so $\alpha \neq 0$

Let $x \in \alpha\overline{\gamma}$ then $x = \alpha w$ such that $w \in \overline{\gamma}$. Consider D a neighbourhood of $\alpha$.Then

$(1/\alpha)$ D is a neighbourhood of w.

Since $w \in \overline{\gamma}$ , $(1/\alpha)\, D \cap \gamma \neq \Phi$

Thus there exists $y \in \gamma$ such that $y = (1/\alpha)\, z$ for some z in D.

There fore $y = (1/\alpha)\, z$ for some $z \in D$.

Also $z = \alpha y \in \alpha\gamma$. Hence $z \in C$. Hence $z \in \alpha\gamma \cap D$.

Therefore $\alpha\gamma \cap D \neq \Phi$.

This shows that any neighbour D of $\alpha w$ intersects $\alpha\gamma$.

Hence $\alpha w \in \overline{\alpha\gamma} \Rightarrow x \in \overline{\alpha\gamma}$.

Thus $x \in \alpha\overline{\gamma} \Rightarrow x \in \overline{\alpha\gamma}$.

Therefore $\alpha\overline{\gamma} \subset \overline{\alpha\gamma}$.

Conversely, Let $w \in \overline{\alpha\gamma}$.

Then $w = \alpha((1/\alpha)w) = \alpha x$ where $x = (1/\alpha)w$.

Let D be a neighbourhood of x, then $\alpha D$ is a neighbourhood of w.

Hence $\alpha D \cap \alpha\gamma \neq \Phi$ .

So there exists an element $z_1 \in \alpha\gamma$.

Hence let $z_1 = \alpha\gamma$ such that $y \in D$ and $y \in \gamma$.

Therefore $y \in D \cap \gamma$.

Thus $D \cap \gamma \neq \Phi$.

But D is a any neighbourhood of x.

Hence $x \in \overline{\gamma}$

Thus $w = \alpha x \in \overline{\gamma}$

Hence $w \in \overline{\alpha\gamma} \Rightarrow w \in \alpha\overline{\gamma}$ . Hence from the above equations

$$\overline{\alpha\gamma} = \alpha\overline{\gamma}$$

Let $\alpha,\beta$ be such that $\alpha \geq 0$, $\beta \geq 0$ and $\alpha+\beta=1$.

Let x, y be elements of $\overline{A}$

Then $\alpha x - \beta y \in \alpha\overline{A} - \beta\overline{A} = \alpha\overline{A} + (-\beta)\overline{A}$ .

Now we are going to show that $(-\beta)\overline{A} = -(\beta\overline{A})$

Let $x \in -(\beta\overline{A})$ then $x = -y$ such that $Y \in \beta\overline{A} = \beta\overline{A}$

Therefore, $y \in \beta\,\overline{A} \Rightarrow y = \beta z$ such that $z \in \overline{A}$ .

Therefore, $x = -\beta z$ such that $z \in \overline{A}$

⇨ $x = -\beta z \in (-\beta)\,\overline{A}$

Therefore, $x \in -(\beta\overline{A}) \Rightarrow x \in (-\beta)\,\overline{A}$.

Conversely, let $x \in (-\beta)\,\overline{A} \Rightarrow x = -\beta y$ such that $y \in \overline{A}$

$\beta y \in \beta\overline{A} = (\beta\overline{A})$

⇨ $-\beta y \in -(\beta\overline{A})$

Therefore $x \in -(\beta\overline{A})$

Therefore $x \in -\beta\,\overline{A} \Rightarrow x \in -(\beta\overline{A})$

Therefore $-\beta\,\overline{A}$ □ $-(\beta\overline{A})$

From the above two relations, we have

$-\beta\,\overline{A} = -(\beta\overline{A})$

Therefore $\alpha x - \beta y \in \alpha\,\overline{A} - \beta\,\overline{A} = \alpha\,\overline{A} + (-\beta)\,\overline{A}$

$= \alpha\,\overline{A} + \overline{(-)}(\beta\overline{A})$

$= \alpha \overline{A} + \overline{-\beta A}$

$= \overline{\alpha A} + \overline{(-\beta A)}$ $\square$ $\overline{\alpha A} + \overline{(-\beta A)} = \overline{\alpha A} + \overline{\beta A}$

Since A is C-Set.

$\alpha A - \beta A = A$

Therefore, $\overline{\alpha A - \beta A} = \overline{A}$

Therefore, $\alpha x - \beta y$ is an element of $\overline{A}$.

This shows that $\overline{A}$ is C- Set.

## References:-

(1) Rudin, W : Functional Analysis, (1973) pp.6-7, P. 11. MC Graw Hill Book Company new York
(2) Kelly, J.L. Namioka, I. and others Linear Topological Spaces, East-West Student Edition, India,1968.