Analytical Solutions of (2+1)-dimensional KdV Equation by Elzaki Transformation

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Abstract

In this work a new Integral transform, namely Elzaki transform was applied to solve (2+1)-dimensional KdV equation to obtain the exact solutions. It is more easier, efficient, simple and powerful tool for solving the equation.

Keywords: Partial Differential Equation, KdV equation, Elzaki Transform.

Introduction

In this paper, we are going to use the newly developed transform namely Elzaki transform and Adomian polynomial in finding the approximate analytical solution to the nonlinear differential equations (to be precise, the KdV equation). It is very important to know the background of these concepts and the assumption on which they are based.

Elzaki Transformation:

Definition : 1

Elzaki Transform of a function f(t) is denoted by E[f(t)] and is defined as follows

$$E[f(t)] = T(v) = v \int_0^\infty e^{\frac{-t}{v}} f(t)dt , \quad \text{where } v \in (-k_1,k_2) \tag{1}$$

We use integration by parts to obtain an Elzaki transformation of partial derivatives as

$$E\left[\frac{\partial f(x,t)}{\partial t}\right] = v \int_0^\infty \frac{\partial f}{\partial t} e^{\frac{t}{v}} dt$$

$$= \lim_{m\to\infty} \int_0^m v \frac{\partial f}{\partial t} e^{\frac{t}{v}} dt .$$

$$= \lim_{m\to\infty} \left\{\left[v e^{\frac{-t}{v}} f(x,t)\right]\right\} - \int_0^m e^{\frac{-t}{v}} f(x,t)\, dt.$$

$$= \frac{T(x,v)}{v} - v f(x,0). \tag{2}$$

We assume that f is piecewise and of exponential order

$E\left[\frac{\partial f(x,t)}{\partial t}\right] = v \int_0^{\infty} \frac{\partial f}{\partial t}\, e^{\frac{t}{v}}\, dt = \frac{\partial}{\partial x} \int_0^{m} v\, e^{\frac{-t}{v}}\, f(x,t)\, dt.$

Using the Leibnitz rule we find that

$E\left[\frac{\partial f}{\partial t}\right] = \frac{d}{dx}[T(x,v)].$

We can also find

$E\left[\frac{\partial^2 f}{\partial x^2}\right] = \frac{d^2}{dx^2}[T(x,v)].$

Elzaki transformation of some functions

S.No	f(t)	g [f(t)] = T(v)
1	1	V^2
2	T	V^3
3	t^n	$n!\, t^{n+2}$
4	e^{at}	$\frac{v^2}{1-av}$
5	te^{at}	$\frac{v^3}{(1-av)^2}$
6	$\frac{t^{n-1}\, e^{at}}{(n-1)!}$	$\frac{v^{n+1}}{(1-av)^n}$
7	Sinat	$\frac{av^3}{1+a^2v^2}$
8	Cosat	$\frac{v^2}{1+a^2v^2}$
9	Sinhat	$\frac{av^3}{1-a^2v^2}$
10	Cos hat	$\frac{av^2}{1-a^2v^2}$
11	e^{at} Sinbt	$\frac{bv^3}{(1-av)^2+b^2v^2}$
12	e^{at} cosbt	$\frac{(1-av)^2v^2}{(1-av)^2+b^2v^2}$

13	t sinat	$\frac{2av^4}{1+a^2v^2}$
14	t cosat	$\frac{v^3}{1+a^2v^2}$

Numerical Applications

Consider the (2+1)- dimensional KdV equation

$u_t + uu_x - uu_{xxx} + u_{xxxxx} + u_{yy} = 0$. (3)

Using an initial condition: $u(x,0) = e^{x+y}$ in the following equation.

$u_t = -[uu_x - uu_{xxx} + u_{xxxxx} + u_{yy}]$.

Now applying the Elzaki transformation we get,

$E[u_t] = -E[uu_x - uu_{xxx} + u_{xxxxx} + u_{yy}]$. (4)

$E[u_t] = \frac{U(x,v)}{v} - vu(x,0)$. (5)

By substituting eqn.(5) in eqn. (4), we obtain that

$\frac{U(x,v)}{v} - vu(x,0) = -E[uu_x - uu_{xxx} + uxxxxx + u_{yy}]$.

$u(x,v) = v^2 e^x - v\,E[uu_x - uu_{xxx} + uxxxxx + u_{yy}]$.

Again by applying the inverse Elzaki transformation we find that,

$u(x,t) = E^{-1}[v^2e^x] - E^{-1}\{ v\,E[uu_x - uu_{xxx} + uxxxxx + u_{yy}] \}$.

$u(x,t) = e^x - E^{-1}\{ v\,E[uu_x - uu_{xxx} + uxxxxx + u_{yy}] \}$. The recursive relation is as follows:

$u_{n+1} = -E^{-1}[A_n + B_n + \frac{\partial^5}{\partial x^5} u_n]$. (6)

Let the nonlinear term be represented by,

$f(u) = u\frac{\partial u}{\partial x} - u\frac{\partial^3 u}{\partial x^3}$.

Let $A_0 = u_0\left[\frac{\partial u_0}{\partial x} - \frac{\partial^3 u_0}{\partial x^3}\right]$,

$A_1 = u_1\left[\frac{\partial u_0}{\partial x} - \frac{\partial^3 u_0}{\partial x^3}\right] + u_0\left[\frac{\partial u_1}{\partial x} - \frac{\partial^3 u_1}{\partial x^3}\right]$.

$$A_2 = u_2\left[\frac{\partial u_0}{\partial x} - \frac{\partial^3 u_0}{\partial x^3}\right] + u_1\left[\frac{\partial u_1}{\partial x} - \frac{\partial^3 u_1}{\partial x^3}\right] + u_0\left[\frac{\partial u_2}{\partial x} - \frac{\partial^3 u_2}{\partial x^3}\right].$$

Then, $B_0 = u_0\left[\frac{\partial u_0}{\partial y} - \frac{\partial^3 u_0}{\partial y^3}\right]$,

$$B_1 = u_1\left[\frac{\partial u_0}{\partial y} - \frac{\partial^3 u_0}{\partial y^3}\right] + u_0\left[\frac{\partial u_1}{\partial y} - \frac{\partial^3 u_1}{\partial y^3}\right] \text{ and}$$

$$B_2 = u_2\left[\frac{\partial u_0}{\partial y} - \frac{\partial^3 u_0}{\partial y^3}\right] + u_1\left[\frac{\partial u_1}{\partial y} - \frac{\partial^3 u_1}{\partial y^3}\right] + u_0\left[\frac{\partial u_2}{\partial y} - \frac{\partial^3 u_2}{\partial y^3}\right].$$

Now substitute n = 0 in eqn.(6), we get

$$u_1 = -E^{-1}\left[v\,E\left\{A_0 + B_0 + \frac{\partial^5}{\partial x^5}u_0\right\}\right]$$

$$= -E^{-1}\left[v\,E\left\{u_0\left[\frac{\partial u_0}{\partial x} - \frac{\partial^3 u_0}{\partial x^3}\right] + u_0\left[\frac{\partial u_0}{\partial y} - \frac{\partial^3 u_0}{\partial y^3}\right] + \frac{\partial^5}{\partial x^5}u_0\right\}\right].$$

$$= -E^{-1}\{v\,E(x+y)\}.$$

$$u_1 = t\,e^{x+y}.$$

$$u_2 = -E^{-1}\left[v\,E\left\{A_1 + B_1 + \frac{\partial^5}{\partial x^5}u_1\right\}\right].$$

$$= -E^{-1}\left[v\,E\left\{u_1\left[\frac{\partial u_0}{\partial x} - \frac{\partial^3 u_0}{\partial x^3}\right] + u_0\left[\frac{\partial u_1}{\partial x} - \frac{\partial^3 u_1}{\partial x^3}\right]\right\} + u_1\left[\frac{\partial u_0}{\partial y} - \frac{\partial^3 u_0}{\partial y^3}\right] + u_0\left[\frac{\partial u_1}{\partial y} - \frac{\partial^3 u_1}{\partial y^3}\right] + \frac{\partial^5}{\partial x^5}u_1\right].$$

$$u_2 = \frac{t^2}{2!}e^{x+y}.$$

Hence the approximate series solution is,

$$u(x,t) = u_0 + u_1 + u_2 + \ldots$$

$$u(x,t) = e^{x+y} + t\,e^{x+y} + \frac{t^2}{2!}e^{x+y} + \ldots$$

$$= e^{x+y}\left[1 + t + \frac{t^2}{2!} + \ldots\right]$$

The Taylor's series expansion of the function, we have

$$e^t = 1 + t + \frac{t^2}{2!} + \cdots.$$

$u(x,t) = e^{x+y} . e^{t}$

$u(x,t) = e^{x+y+t}$.

Conclusion

(i) In this research work, Elzaki transformation is used to solve the (2+1)-dimensional KdV equation.

(ii) This result has been extracted that Elzaki transform plays a key role in finding the analytical solution for a wide class of initial value problems.

(iii) It is heavily used in solving PDEs and ODEs. The implementation of this method shows that it is easy to use, simple, time saving than the other existing methods.

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