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APPROXIMATE ANALYTICAL SOLUTION OF DRUG ADDICTION USING HOMOTOPY PERTURBATION METHOD

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Abstract : In this paper we are dealing with the mathematical modelling and approximate analytical solutions for drug addiction model. Analytical expressions for drug addiction concentrations are derived using Homotopy perturbation method. In this paper , the analytical results have beencompared with perturbation method and which showed that, the present approach has less computational aspect and are applicable for solving other non-linear boundary value problem.An excellent agreement with simulation is observed.

IndexTerms - Non-Linear Equations, Mathematical Modelling, Drug Diffusion, HomotopyPerturbation Method, Numerical Simulation.

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I. INTRODUCTION

Drug addiction can start with experimental use of a recreational drug in social situations, and for some people, the drug use becomes more frequent. For others, particularly with opioids, drug addiction begins with exposure to prescribed medications, or receiving medications from a friend or relative who has been prescribed the medication .

The risk of addiction and how fast you become addicted varies by drug. Some drugs, such asopioid painkillers, have a higher risk and cause addiction more quickly than others. Developing an addiction to drugs isn't a character flaw or a sign of weakness, and it takes more than willpower to overcome the problem. Abusing illegal or certain prescription drugs can create changes in the brain, causing powerful cravings and a compulsion to use that makes sobriety seem like an impossible goal. But recovery is never out of reach, no matter how hopeless your situation seems or how many times you've tried and failed before. With the right treatment and support, change is always possible([1,2,3,4]). For many people struggling with addiction, the toughest step toward recovery is the very first one: recognizing that you have a problem and deciding to make a change. It's normal to feel uncertain about whether you're ready to start recovery, or if you have what it takes to quit. The mathematical model plays an important role in understanding the behavior of epidemics and biology ([5, 6, 7]). Drug abuse has almost identical characteristics with conventional epidemics. So far, various mathematical models describing the spread of drug abuse has been carried out (see [8, 9, 10, 11]). Bobashev et al. (2017) have created an abstract nonlinear dynamical model of smoking based on the opponent-process and the allostatic theory. It's also normal to feel conflicted about giving up your drug of choice, even when you know it's causing problems in your life. Recovery requires time, motivation, and support, but by making a commitment to change, you can overcome your addiction and regain control of your life. In this paper , we have derived analytical expressions for concentration using homotopy perturbation method. The solution of the system remains positive at any time if theinitial values of all the variables are all positive.

II. MATHEMATICAL FORMULATION OF THE PROBLEM :

In the region under consideration ,the total population is divided into four classes: The susceptible population S(t), the addicted population A(t) ,the treatment class T(t) and theremoved class R(t). Let M(t) be the cumulative density of the awareness programs being implemented in that region at time t.

The model of drug addiction can be represented as the following concentrations

$$\frac{ds}{dt} = A - \alpha SA + (1-\beta) - PMS - \mu S \quad (2.1)$$

$$\frac{dA}{dt} = \alpha SA - qAM - \mu A - vA \quad (2.2)$$

$$\frac{dT}{dt} = qAM - \theta T - \mu T \quad (2.3)$$

$$\frac{dR}{dt} = PMS + \beta\theta T - \mu R \quad (2.4)$$

$$\frac{dM}{dt} = kA - \delta M \quad (2.5)$$

Where S(t) represents the number of susceptible population, A(t)) represents the number ofaddicted population, T(t)) represents the number of treatment class, R(t)) represents the number of removed class and M(t) cumulative density. The susceptible class comprises of

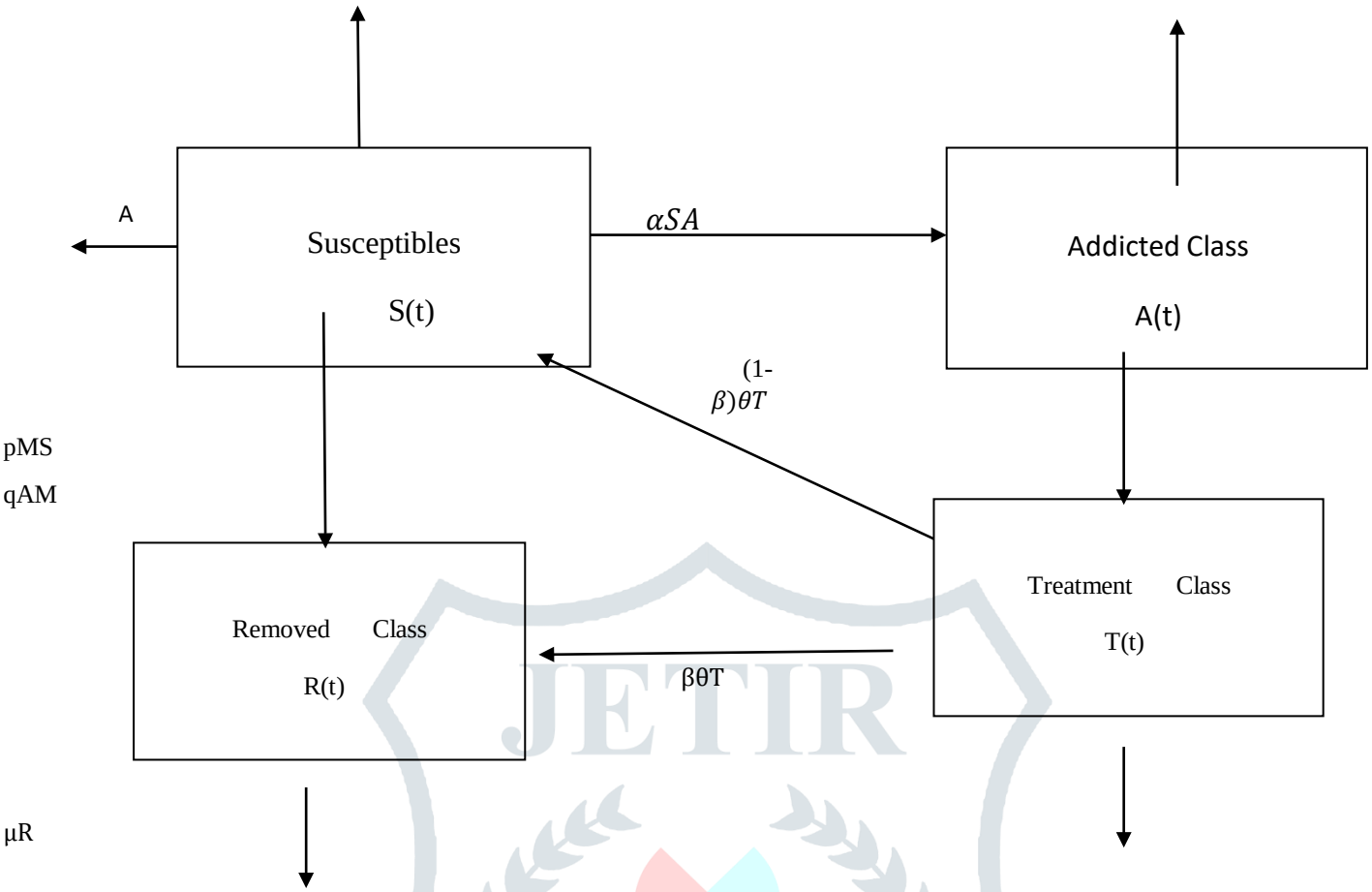
potential drug users who could be immigrants or individuals returing after treatment. The addicted population refers to the group of people actively involved in drug usage. The treatment class consists of individuals coming from the addicted community to undertake treatment due to the influence of awareness programs. The removed class is made up of alert individuals who avoid contact with the addicted individuals due to their awareness about theissue.

The initial conditions for the above equation as t=0 are $S = S_i; = A_i; T = T_i; R = R_i;$

$M = M_i.$

The following assumptions have been made in the formulation of the model.

1. Addiction occurs only due to contact between the susceptibles and addicted individuals.
2. The growth density of awareness programs is linearly proportional to the number ofaddicted individuals.
3. Individuals in the removed class avoid contact with the addicted population and aretherefore safe from addiction.
4. Only a fraction of the individuals exposed to the awareness programs choose to isolatethemselves form the addicted group.
5. Individuals move form the addicted class to the treatment class due to the impact of theawareness programs only. Hence ,all individuals in the treatment class are aware.
6. During the treatment phase, an individual doesn't come in contact with the addictedcommunity.
7. Individuals in the treatment class remain influenced by the awareness programs and onlythose with strong will power to quit the addiction move to the removed class, while the restrevert back to the susceptible class.



NOMENCLATURE:

Γ	Time
A	Constant rate of immigration
u	Natural death rate
V	Drug induced death rate
P	Proportion of aware individuals in susceptible class moving to the removed class
Q	Proportion of aware individuals in the addicted class moving to the treatment class
β	Proportion of treated individuals having strong determination (to quit addiction permanently)
θ	Outflow rate from treatment class
α	Contact rate of susceptible and individuals

K	Proportionality factor between awareness programs density and number of addicted individuals (measure of the rate at which awareness programs are being implemented)
δ	Depletion rate of these programs (due to ineffectiveness, social problems in population etc.)

III Solution of the Problem Using Homotopy Perturbation Method

Linear and non-linear phenomena are of fundamental importance in various fields of scienceand engineering. Most models of real – life problems are still very difficult to solve. Therefore, approximate analytical solutions such as Homotopy perturbation method (HPM) ([12-25]) were introduced. This method is the most effective and convenient ones for both linear and non-linear equations. Perturbation method is based on assuming a small parameter.The majority of non-linear problems, especially those having strong non-linearity, have no small parameters at all and the approximate solutions obtained by the perturbation methods, in most cases, are valid only for small values of the small parameter. Generally, the perturbation solutions are uniformly valid as long as a scientific system parameter is small. However, we cannot rely fully on the approximations, because there is no criterion on whichthe small parameter should exists. Thus, it is essential to check the validity of the approximations numerically and/or experimentally. To overcome these difficulties, HPM have been proposed recently.

Recently, many authors have applied the Homotopy perturbation method (HPM) to solve the non-linear problem in physics and engineering sciences ([12-15]). This method is also used tosolve some of the non-linear problem in physical sciences ([16-18]). This method is a combination of Homotopy in topology and classic perturbation techniques. Ji-Huan He used to solve the Lighthill equation [16], the Duffing equation [17] and the Blasius equation ([18- 19]). The HPM is unique in its applicability, accuracy and efficiency. The HPM uses the imbedding parameter and only a few iterations are needed to search for an asymptotic solution. Using this method([20-25]), (Appendix.A)Supposing the approximate solutions of eqn.(2.1-2.5) have the form

$$\frac{ds}{dt} = A - \alpha SA + (1-\beta) - PMS - \mu S$$

$$S_1 = e^{-vt}\left[\frac{A_i}{(-v)} - \alpha S_i A_i \frac{e^{-(\mu+v)t}}{e^{\mu t}(-\mu-v)} + (1-\beta) T_i \frac{e^{-\theta t}}{e^{\mu t}(-\theta)}\right] + \frac{C}{e^{\mu t}} \quad (3.1)$$

Put $S_1 = 0, t = 0$

$$0 = -\frac{S_i A_i}{(-v)(-\mu-v)} + \frac{(1-\beta)\theta T_i}{(-\theta)} + C$$

$$C = \frac{S_i A_i}{(-v)(-\mu-v)} - \frac{(1-\beta)\theta T_i}{(-\theta)}$$

$$S_1 = \frac{e^{-vt}}{(-v)} A_i - \alpha S_i A_i \frac{e^{-(\mu+v)t}}{e^{\mu t}(-\mu-v)} + (1-\beta) T_i \frac{e^{-\theta t}}{e^{\mu t}(-\theta)} - \frac{A_i}{(-v)e^{\mu t}} + \frac{\alpha S_i A_i}{(-\mu-v)e^{\mu t}} - \frac{(1-\beta)\theta T_i}{(-\theta)e^{\mu t}} \quad (3.2)$$

$$\frac{dA}{dt} = \alpha SA - qAM - \mu A - vA$$

$$A_1 \frac{A_i \alpha S_i e^{-\mu t}}{e^{(\mu+v)t}(-\mu)} \frac{A_i q M_i e^{-\delta t}}{e^{(\mu+v)t}(-\delta)} \frac{C}{e^{(\mu+v)t}} \quad (3.3)$$

Put $A_1 = 0, t = 0$

$$0 = \frac{A_i \alpha S_i}{(-\mu)} - \frac{A_i}{(-\delta)} + C$$

$$C = -\frac{A_i \alpha S_i}{(-\mu)} + \frac{A_i q M_i}{(-\delta)}$$

$$A = \frac{A_i \alpha S_i e^{-\mu t}}{(\mu+v)(-\mu)} - \frac{A_i q M_i e^{-\delta t}}{e^{(\mu+v)t}(-\delta)} + \frac{A_i q M_i}{(-\delta)e^{(\mu+v)t}} - \frac{A_i \alpha S_i}{(-\mu)e^{(\mu+v)t}} \quad (3.4)$$

$$\frac{dT}{dt} = qAM - \theta T - \mu T$$

$$T_1 = qA_i \left[\frac{e^{-(\mu+v)t}}{-(\mu+v)}\right]\left[\frac{e^{-\delta t} M_i}{-\delta}\right] + \frac{C_3}{e^{(\theta+\mu)t}} \quad (3.5)$$

Put $T_i = 0, t = 0$

$$C_3 = -\left[\frac{qA_i}{-(\mu+v)}\right]\left[\frac{M_i}{-\delta}\right]$$

$$T_1 = qA_i\left(\frac{e^{-(\mu+v)t}}{-(\mu+v)}\right)\left(\frac{e^{-\delta t}M_i}{-\delta-(\mu+v)(-\delta)e^{(\theta+\mu)t}}\right) - \frac{qA_iM_i}{} \quad (3.6)$$

$$\frac{dR}{dt} = PMS + \beta\theta T - \mu R$$

$$R_1 = \frac{\beta\theta T_i e^{-\theta t}}{(-\theta)(e^{\mu t})} + \frac{C_4}{e^{\mu t}}$$

Put $R_1 = 0, t = 0$

$$C_4 = \frac{-\beta\theta T_i}{-\theta}$$

$$R_1 = \frac{\beta\theta T_i e^{-\theta t}}{(-\theta)(e^{\mu t})} - \frac{\beta\theta T_i}{(-\theta)e^{\mu t}} \quad (3.7)$$

$$\frac{dM}{dt} = kA - \delta M$$

$$M_1 = \frac{k^{(-\mu-v)t}}{(\delta-\mu-v)} + \frac{C_5}{e^{\delta t}} \quad (3.8)$$

Put $M_1 = 0, t = 0$

$$C_5 = \frac{-kA_i}{(\delta-\mu-v)}$$

$$M_1 = \frac{kA_i}{(\delta-\mu-v)}\left((-\mu-v) - 1\right) \quad (3.9)$$

Susceptible population are given by

$$S_1 = e^{-vt}\left(\frac{A}{(-v)} - \alpha S_i A_i\left(\frac{e^{-(\mu+v)t}}{e^{\mu t}(-\mu-v)} + (1-\beta)\right)\left(\frac{e^{-\theta t}}{e^{\mu t}(-\theta)} - \frac{A}{(-v)e^{\mu t}}\right) + \frac{\alpha S_i A_i}{(-\mu-v)e^{\mu t}} - \frac{(1-\beta)}{(-\theta)e^{\mu t}}\right) \quad (3)$$

Addicted population are given by

$$A_1 = \frac{A_i\alpha S_i e^{-\mu t}}{(\mu+v)(-\mu)} - \frac{A_i q M_i e^{-\delta t}}{e^{(\mu+v)t}(-\delta)} + \frac{A_i q M_i}{(-\delta)e^{(\mu+v)t}} - \frac{A_i\alpha S_i}{(-\mu)e^{(\mu+v)t}} \quad (4)$$

Treatment class are given by

$$T_1 = qA_i\left(\frac{e^{-(\mu+v)t}}{-(\mu+v)}\right)\left(\frac{e^{-\delta t}M_i}{-\delta-(\mu+v)(-\delta)e^{(\theta+\mu)t}}\right) - \frac{qA_iM_i}{} \quad (5)$$

Removed class are given by

$$R_1 = \frac{\beta\theta T_i e^{-}}{(-\theta)(e^{\mu t})} - \frac{\beta\theta T_i}{(-\theta)e^{\mu t}} \quad (6)$$

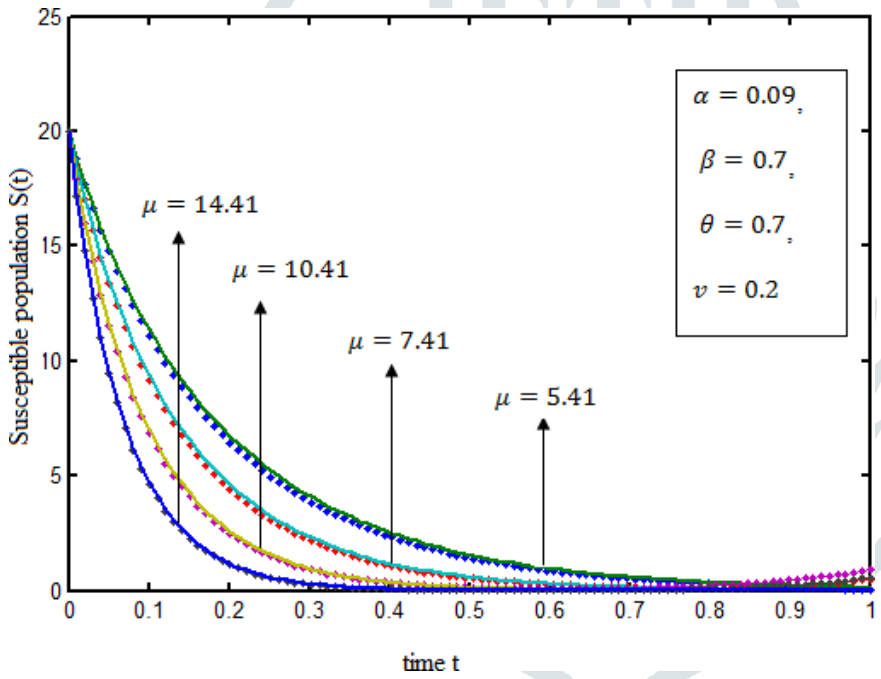
Cummulative density are given by

$$M_1 = \frac{kA_i}{(\delta-\mu-v)}\left((-\mu-v) - 1\right) \quad (7)$$

IV. RESULTS AND DISCUSSION

Eqns.(3-7) are the new and simple approximate analytical expressions for the concentrationslike Susceptible population, Addicted population, Treatment class, Removed class, Cummulative density.

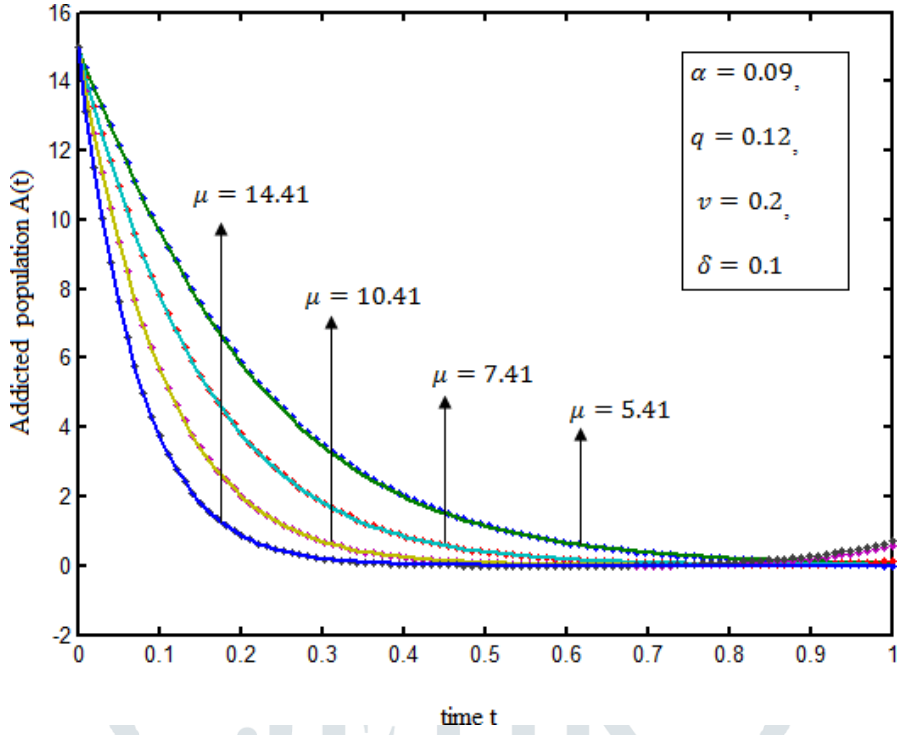
FiguresandTables



Figure(4.1)

Figure(4.1): Plot of Susceptible Population S(t) versus Time(t). The ratio is computed byusing different values of parameter μ and some fixed values of parametersα=0.09, β=0.7, θ=0.7, v=0.2 in equation(2.1).

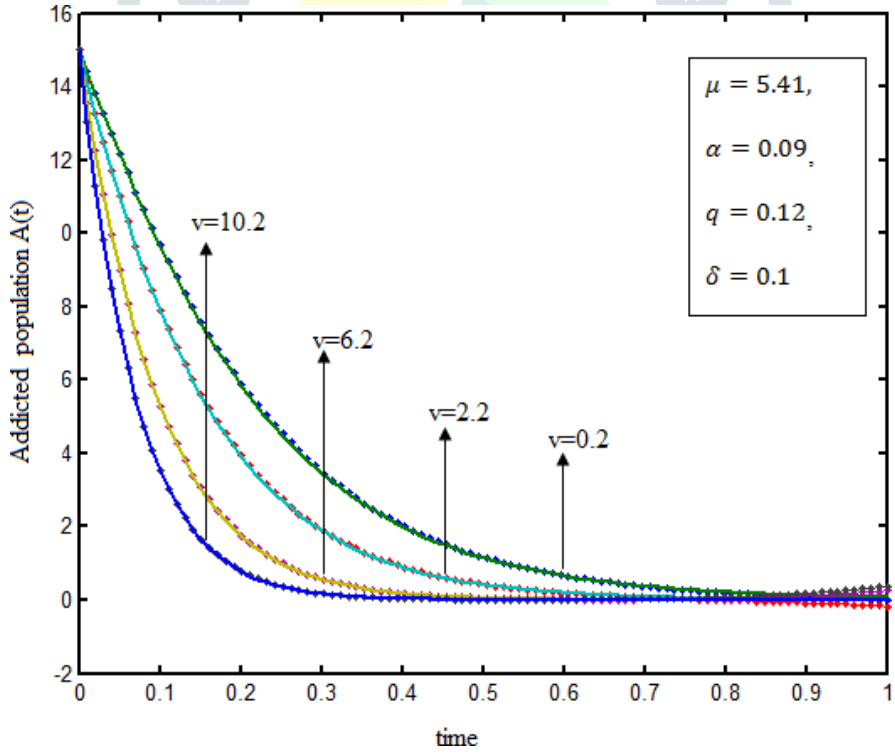
The key to the graph: solid line represents the Equation(2.1) and dotted line represents thenumerical solution. **Figure(4.1)** represents the Susceptible Population S(t) versus Time(t) by using different values of parameter μ and some fixed values of parameters α=0.09, β=0.7, θ=0.7, v=0.2 in equation(2.1). We observe that the ratio of Susceptible population S(t) is gradually decreasingwhen time decreases.



Figure(4.2)

Figure(4.2): Plot of Addicted Population A(t) versus Time(t). The ratio is computed by usingdifferent values of parameter μ and some fixed values of parameters α=0.09, q=0.12, v=0.2, δ=0.2 in equation(2.2).

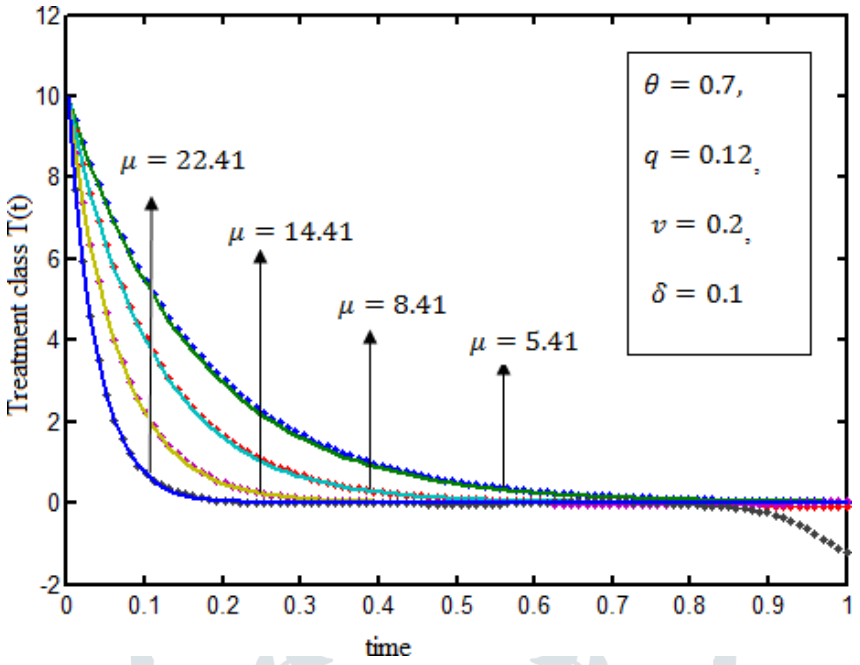
The key to the graph: solid line represents the Equation(2.2) and dotted line represents thenumerical solution.**Figure(4.2)** represents the Addicted Population A(t) versus Time(t) by using different valuesof parameter μ and some fixed values of parameters α=0.09, q=0.12, v=0.2, δ=0.2 in equation(2.2). We observe that the ratio of Addicted population A(t) is gradually decreasing when time decreases.



Figure(4.3)

Figure(4.3): Plot of Addicted Population A(t) versus Time(t). The ratio is computed by usingdifferent values of parameter v and some fixed values of parameters α=0.09, μ=5.41, q=0.12,δ=0.1 in equation (2.2).

The key to the graph: solid line represents the Equation(2.2) and dotted line represents thenumerical solution. **Figure(4.3)** represents the Addicted Population A(t) versus Time(t) by using different valuesof parameter v and some fixed values of parameters α=0.09, μ=5.41, q=0.12, δ=0.1 in equation (2.2).We observe that the ratio of Addicted population A(t) is gradually decreasing when time decreases.

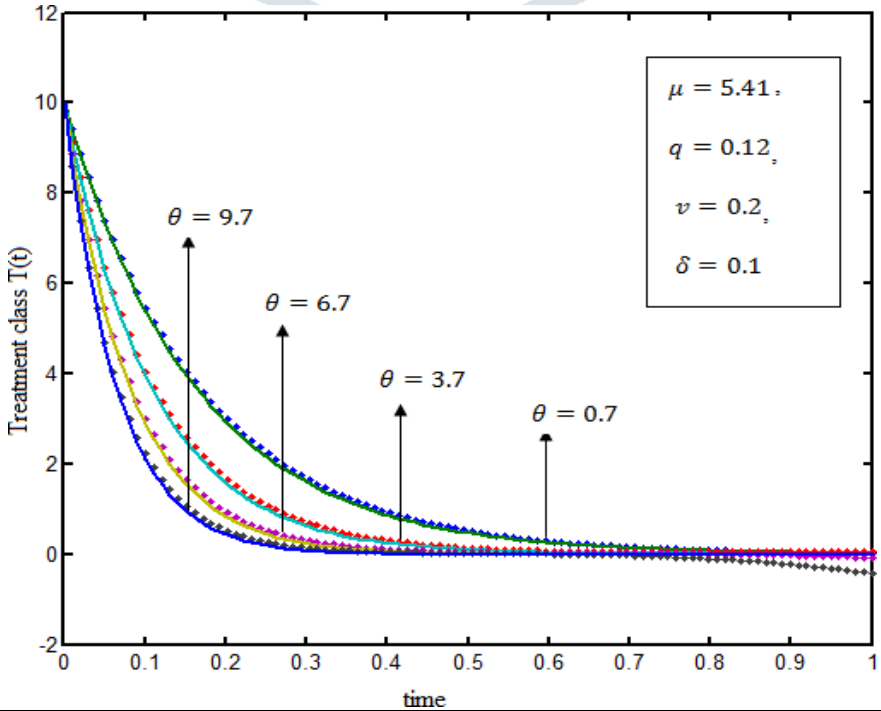


Figure(4.4)

Figure(4.4): Plot of Treatment class T(t) versus Time(t). The ratio is computed by using different values of parameter μ and some fixed values of parameters $\theta = 0.7$, q=0.12, v=0.2,$\delta = 0.1$ in equation(2.3).

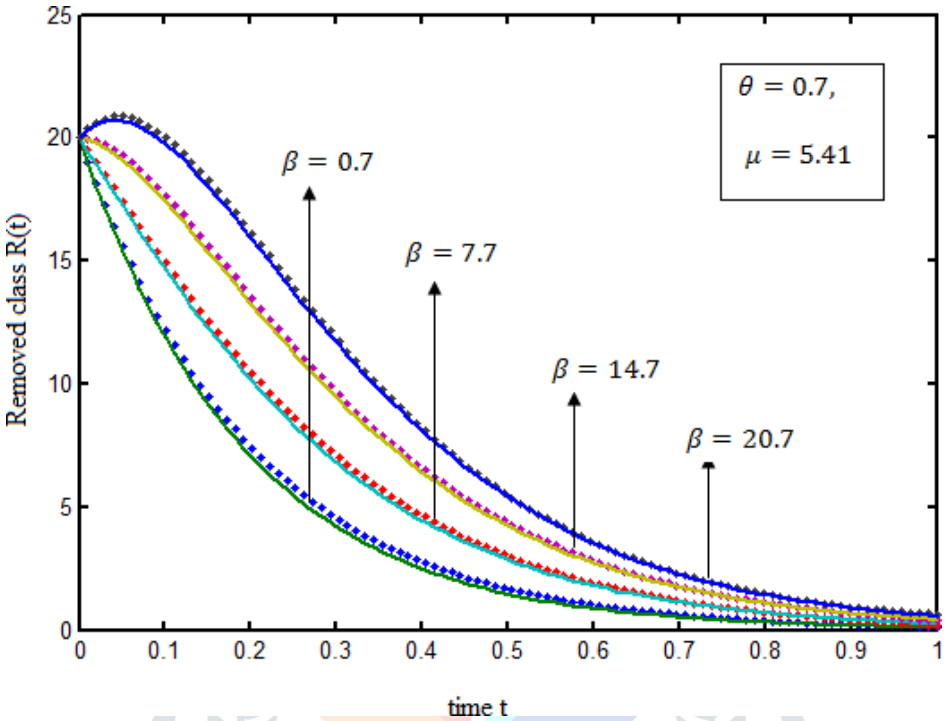
The key to the graph: solid line represents the Equation(2.3) and dotted line represents thenumerical solution. **Figure(4.4)** represents the Treatment class T(t) versus Time(t) by using different values of parameter μ and some fixed values of parameters $\theta = 0.7$, q=0.12, v=0.2, $\delta = 0.1$ in equation(2.3). We observe that the ratio of Treatment class T(t) is gradually decreasing when time decreases.

Figure(4.5)



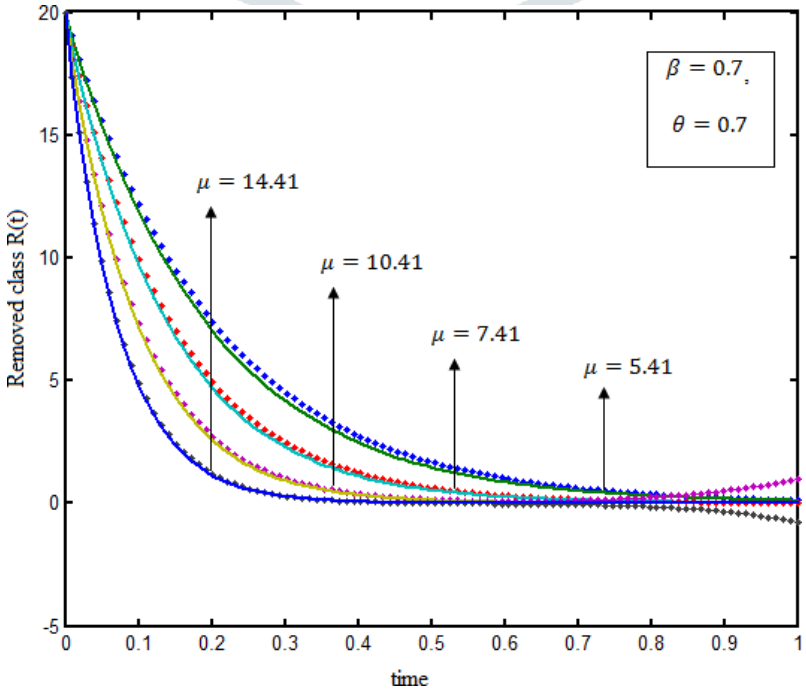
Figure(4.5): Plot of Treatment class T(t) versus Time(t). The ratio is computed by usingdifferent values of parameter θ and some fixed values of parameters $\mu = 5.41$, q=0.12, v=0.2, $\delta = 0.1$ in equation(2.3).

The key to the graph: solid line represents the Equation(2.3) and dotted line represents thenumerical solution.**Figure(4.5)** represents the Treatment class T(t) versus Time(t) by using different values of parameter θ and some fixed values of parameters $\mu = 5.41$, q=0.12, v=0.2, $\delta = 0.1$ in equation(2.3). We observe that the ratio of Treatment class T(t) is gradually decreasing when time decreases.



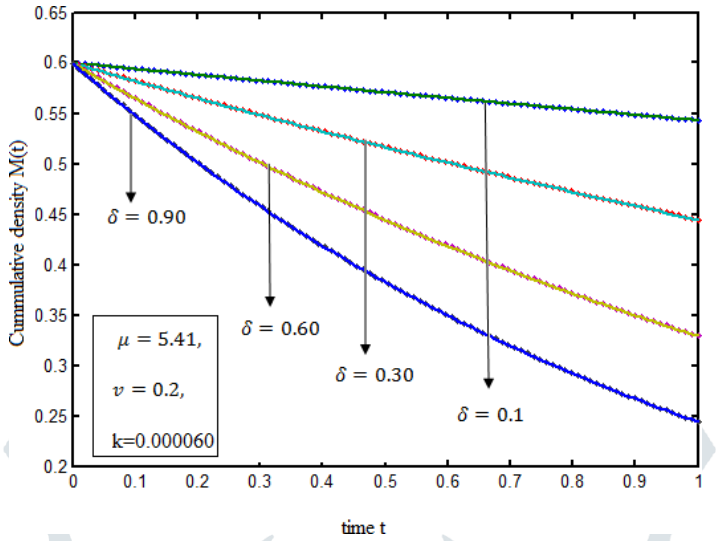
Figure(4.6)

Figure(4.6): Plot of Removed class R(t) versus Time(t). The ratio is computed by using different values of parameter β and some fixed values of parameters $\mu = 5.41$, $\theta = 0.7$ inequation(2.4).The key to the graph: solid line represents the Equation(2.4) and dotted line represents thenumerical solution.**Figure(4.6)** represents the Removed class R(t) versus Time(t) by using different values ofparameter β and some fixed values of parameters $\mu = 5.41$, $\theta = 0.7$ in equation(2.4). We observe that the ratio of Removed class R(t) is gradually increasing when time increases.



Figure(4.7)

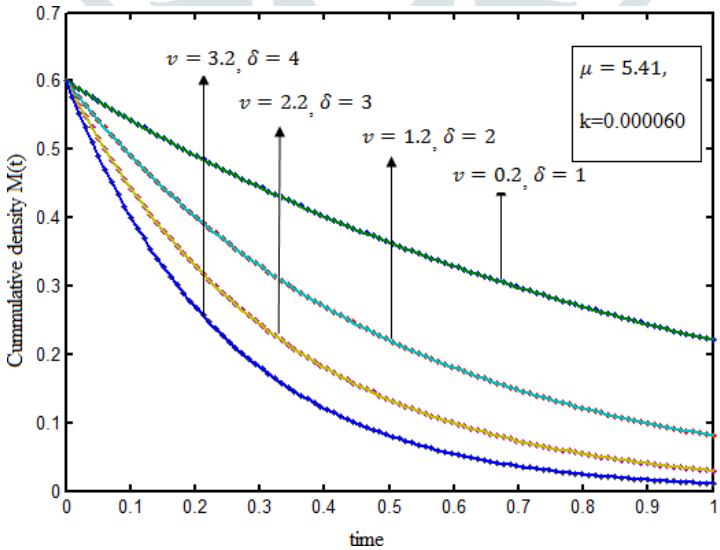
Figure(4.7): Plot of Removed class R(t) versus Time(t). The ratio is computed by using different values of parameter μ and some fixed values of parameters $\beta = 0.7, \theta = 0.7$ in equation(2.4).The key to the graph: solid line represents the Equation(2.4) and dotted line represents the numerical solution.**Figure(4.7)** represents the Removed class R(t) versus Time(t) by using different values of parameter μ and some fixed values of parameters $\beta = 0.7, \theta = 0.7$ in equation (2.4). We observe that the ratio of Removed class R(t) is gradually decreasing when time decreases.



Figure(4.8)

Figure(4.8): Plot of Cummulative density M(t) versus Time(t). The ratio is computed by using different values of parameter δ and some fixed values of parameters μ=5.41, v=0.2,k=0.000060 in equation(2.5).

The key to the graph: solid line represents the Equation(2.5) and dotted line represents thenumerical solution.**Figure(4.8)** represents the Cummulative density M(t) versus Time(t) by using different values of parameter δ and some fixed values of parameters μ=5.41, v=0.2, k=0.000060 in equation(2.5). We observe that the ratio of cumulative density M(t) is gradually decreasing when time increases.



Figure(4.9)

Figure(4.9): Plot of Cummulative density M(t) versus Time(t). The ratio is computed byusing different values of parameter δ and v and some fixed values of parameters μ=5.41, k=0.000060 in equation(2.5).The key to the graph: solid line represents the Equation(2.5) and dotted line represents thenumerical solution.**Figure(4.9)** represents the Cummulative density M(t) versus Time(t)

by using different values of parameter δ and v and some fixed values of parameters μ=5.41, k=0.000060 in equation(2.5). We observe that the ratio of cumulative density M(t) is gradually decreasingwhen time increases.

V. NUMERICAL SIMULATION:

The non-linear differential equation(2.1)-(2.5) for the given initial conditions are solved by using homotopy perturbation method . software. Its numerical solution is compared with Homotopy perturbation method in Figs (4.1) – (4.9) and it gives satisfactory result for all small values of the parameters. The Matlab program is The function pdex 4 in matlabs software, which is a function of solving boundary value problems was used to solvethese equations numerically.

VI. CONCLUSION:

The time dependent system of non-linear differential equations in drug addiction model have been solved analytically and numerically. In the present study, analytical expression corresponding to concentration profile in terms of all parameters is presented. It is observed that the results obtained using the analytical expression show a good conformitywith numerical simulation results. Thus the analytical results helps us to understand the effect of various parameters on the drug addiction model.

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