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FINITE DIMENSIONAL LIE ALGEBRAS OF VECTOR FIELDS

LIE ALGEBRAS

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Abstract
Algebra is a vector space over a field equipped with an associative, bilinear multiplication. This forces us to examine properties of ring theory under the lens oflinear algebra, solvability, as we determine subalgebras, ideals. We completely classify all
„Lie theory and special functions". Applications to finite-dimensional Lie algebra in problems are indicated. A homogeneous space approach is used to investigate of Liealgebras of vector fields with coefficients in smooth functions. By use of a new bilinear operation, any associative algebra into a non associative abstract Lie algebra, where the new product in terms of the given associative product is the commutator. Finite- dimensional real analytic Lie algebras of vector fields on $\mathbb{R}^2$ are completely classified up tochanges of local coordinates. Lie algebras of first-order differential operators in two complex variables. We completely classify all „Lie algebras" in two complex and real variables

IndexTerms –
Lie algebra, Lie groups, Subalgebras, Cohomology, Isomorphism, Two complex and real variables, finite-dimensional, Lie theory, Lie algebras of vector field, first-order differential operators.

LEMMA

If g is a quasi-exactly solvable Lie algebra of differential operatorsdescribed by the triple [□, m, [F]], then either m = 0 or m = {1}.

PROOF:

Suppose □$(x) \in m$ and $0 \neq$ g $(x) \in n$, the finite-dimensional module for g. Then since h defines a multiplication operator in g, we must have □(x) g$(x) \in n$ also. Iterating the multiplication operator, we deduce that □$(x)^n$ g$(x) \in n$ for any power n ≥ 0. But n is finite-dimensional, so taking n greater than the dimension of n, we deduce a linear dependency of the form $\sum c_k$ □$(x)^k$ g(x) = 0, for constants c_k . Since g$(x) \neq 0$, □ must satisfy a constant coefficient polynomial equation $\sum c_k$ □$(x)^k$ = 0.
We conclude that □(x) itself is a constant.

THEOREM

If g is a locally primitive Lie algebra of vector fields in $\mathbb{R}^2$ not
locally equivalent under a real analytic change of coordinates to one of the algebras, we can choose local coordinates around a point P generic for g such that $g - g^2$ is generated

P

PROOF:

$p + \cdots, q + \cdots, \alpha(xp + yq) + (yp - xq) + \cdots$

$p + \cdots, q + \cdots, xp + yq + \cdots, yp - xq + \cdots$

First of all, g contains two vector fields linearly independent at P. By taking
suitable linear combinations of these two vector fields, we see that g contains the vectorfields
$p + \cdots, q + \cdots,$

where the dots are terms of order 1 or higher at P. The latter vector fields are a basis for thevector fields of zero at P, that is, their equivalence classes modulo g_P are a basis for g/g_P . Finally, since

$g/g^2 \approx (g/g_P)$ $(g/g^2) \equiv (g/g_P)$ $f_P(g)$,

P P

The theorem follows from the results about $f_P(g)$ obtained above.

We shall determine in this section all the equivalence classes of locally primitive non-semisimple Lie algebras of vector fields in $\mathbb{R}^2$ not already included in Lie‟s list.

THEOREM

Let g be a non-semisimple locally primitive Lie algebra of vector fieldsin $\mathbb{R}^2$ not locally equivalent to one of the algebras. Then there are local coordinates (x, y) in which either

(i) g = span $\{p, q, \alpha(xp + yq) + yp - xq\}$

for some $\alpha \in \mathbb{R}$, or

(ii) g = span $\{p, q, xp + yq, yp - xq$

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