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On Some New Sequence Spaces of Non-Absolute Type Related to ℓ_p^λ and ℓ_∞^λ

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Abstract : In this paper, we introduce the sequence spaces $\ell_p^\lambda(\Delta)$ and $\ell_\infty^\lambda(\Delta)$ of non-absolute type and prove that the spaces $\ell_p^\lambda(\Delta)$ and $\ell_\infty^\lambda(\Delta)$ are linearly isomorphic for $0 < p < \infty$. We show that $\ell_p^\lambda(\Delta)$ is a p −normed space for $0 < p < 1$ and BK-space for $1 \le p < \infty$. Further, we derive some inclusions on $\ell_p^\lambda(\Delta)$. Finally, we construct a basis for $\ell_p^\lambda(\Delta)$.

IndexTerms - Sequence spaces, convergence, boundedness, linear isomorphism.

I. INTRODUCTION.

By $\{t_n\}$, we denote the sequence, where $t_n \in \mathbb{R}\ or\ \mathbb{C}$ for all $n \in \mathbb{N}$. A universal sequence space $(w, +, .)$,

$$w = \{\{t_n\}: t_n \in \mathbb{R}\ or\ \mathbb{C}\ \forall\ n \in \mathbb{N}\}$$
$$\ell_p = \{\{t_n\} \in w: \}$$
$$\ell_\infty = \{\{t_n\} \in w: |t_n| \le K\ for\ some\ K \in [0, \infty)\}$$
$$c_0 = \{\{t_n\} \in w: t_n \to 0\}$$
$$c = \left\{\{t_n\} \in w: t_n \to \alpha,\ \ \alpha \in \mathbb{R}\ or\ \mathbb{C}\ according\ t_n \in \mathbb{R}\ or\ \mathbb{C}\ \forall\ n \in \mathbb{N}\right\}$$

$\ell_p, \ell_\infty, c_0, c$ are subspaces of w. A sequence space Ω with linear topology is said to be K −space if the function $\gamma_k: \Omega \to \mathbb{C}$, $\gamma_k(\{t_n\}) = t_k$ is continuous $\forall k \in \mathbb{N}$. If a K −space Ω is a complete linear metric space then it is called as FK −space further a FK −space with normable topology is known as BK −space (Nanda, 1989). ℓ_∞, c_0, c are BK −spaces with respect to the norm $||\{t_n\}|| = \sup_n |t_n|$. Also ℓ_p is a $BK - space$ (Maddox, 1988)with norm,

$$||\{t_n\}||_{\ell_p} = \begin{cases} \sum_n |t_n|^p & ; \quad p \in (0,1) \\ \left(\sum_n |t_n|^p\right)^{\frac{1}{p}} & ; \quad p \in [1, \infty) \end{cases}$$

Let $\mathcal{M} = [\alpha_{pq}]$ be an infinite matrix, $\alpha_{pq} \in \mathbb{R}\ or\ \mathbb{C}$, $p, q \in \mathbb{N}$. Then $\mathcal{M}$ defines a matrix mapping from a sequence space Ω to sequence space Θ if for each $t = \{t_q\} \in \Omega$ the sequence $\mathcal{M}t = \{\mathcal{M}_p(t)\}$, where

$$\mathcal{M}_p(t) = \sum_q \alpha_{pq} t_q$$

The family of all $\mathcal{M}'s$ that map Ω into Θ is denoted by $(\Omega: \Theta)$. For a sequence space Ω, the domain of $\mathcal{M}$ in Ω is defined as

$$\Omega_{\mathcal{M}} = \{t \in w : \mathcal{M}t \in \Omega\}$$

$\Omega_{\mathcal{M}}$ itself a sequence space.

Constructing a sequence space by means of matrix domain has been employed by many authors. The approach of constructing a new sequence space by means of the matrix domain of a particular limitation method has recently been employed by several authors, e.g., Wang [19], Ng and Lee [18], Malkowsky [12], Başar and Altay [7], Malkowsky and Savaş [13], Aydın and Başar [3, 4, 5, 6], Altay and Başar [1], Altay, Başar and Mursaleen [2, 14] and Mursaleen and Noman [15, 16], respectively. They introduced the sequence spaces $(\ell_\infty)_{N_q}$ and c_{N_q} in [19], $(\ell_\infty)_{C_1} = X_\infty$ and $(\ell_p)_{C_1} = X_p$ in [18], $(\ell_\infty)_{R^t} = r_\infty^t, c_{R^t} = r_c^t$ and $(c_0)_{R^t} = r_0^t$ in [12], $(\ell_p)_\Delta = bv_p$ in [7], $\mu_G = Z(u,v;\mu)$ in [13], $(c_0)_{A^r} = a_0^r$ and $c_{A^r} = a_c^r$ in [3], $[c_0(u,p)]_{A^r} = a_0^r(u,p)$ and $[c(u,p)]_{A^r} = a_c^r(u,p)$ in [4], $(a_0^r)_\Delta = a_0^r(\Delta)$ and $(a_c^r)_\Delta = a_c^r(\Delta)$ in [5], $(\ell_p)_{A^r} = a_p^r$ and $(\ell_\infty)_{A^r} = a_\infty^r$ in [6], $(c_0)_{E^r} = e_0^r$ and $c_{E^r} = e_c^r$ in [1], $(\ell_p)_{E^r} = e_p^r$ and $(\ell_\infty)_{E^r} = e_\infty^r$ in [2,14], $(c_0)_\Lambda = c_0^\lambda$ and $c_\Lambda = c^\lambda$ in [15] and $(c_0^\lambda)_\Delta = c_0^\lambda(\Delta)$ and $(c^\lambda)_\Delta = c^\lambda(\Delta)$ in [16], where N_q, C_1, R^t and E^r denote the Nörlund, Cesàro, Riesz and Euler means, respectively, Δ denotes the band matrix defining the difference operator, G and A^r are defined in [13] and [3], respectively, Λ is defined in Section 2, below, $\mu \in \{c_0, c, \ell_p\}$ and $1 \le p < \infty$. Also $c_0(u,p)$ and $c(u,p)$ denote the sequence spaces generated from the Maddox's spaces $c_0(p)$ and $c(p)$ by Başarı [8]. Verma[22] introduced the soft real sequences. Noman [21] introduced the sequence spaces ℓ_p^λ and ℓ_∞^λ of non-absolute type. The Main purpose of this paper is to introduce the sequence spaces $\ell_p^\lambda(\Delta)$ and $\ell_\infty^\lambda(\Delta)$ of non-absolute type and to derive some results.

II. $\boldsymbol{\lambda}$-boundedness and $\boldsymbol{p}$-absolute convergence of type $\boldsymbol{\lambda}$

Suppose, $\lambda = \{\lambda_k\}, k \in 0,1,2,\ldots$ be a sequence such that $0 < \lambda_k < \lambda_{k+1}$diverges to ∞. Define,

$$\Lambda_n(t) = \frac{1}{\lambda_n}\sum_{k=0}^{n}(\lambda_k - \lambda_{k-1})t_k$$

$\lambda_{-1} = 0,\ n \in \mathbb{N}$. A sequence $t = \{t_k\} \in w$ is said to be λ-bounded [21] if $\sup_n |\Lambda_n(t)| \in \mathbb{R}$. Also the series $\sum_k t_k$ is p-absolutely,$p \in (0,\infty)$, convergent of type λ if $\sum_n |\Lambda_n(t)|^p$ converges to real number.

Lemma 2.1. A sequence $t = \{t_k\} \in \ell_\infty$ implies t is λ-bounded.
Define an infinite matrix $\Lambda = [\lambda_{nk}]_{n,k=0,1,2,\ldots}$ by

$$\lambda_{nk} = \begin{cases} \dfrac{\lambda_k - \lambda_{k-1}}{\lambda_n} & ; k \le n \\ 0 & ; k > n \end{cases}$$

Then, for $t = \{t_k\} \in w$, the $\Lambda-$transform of t is given by $\Lambda(t) = \{\Lambda_n(t)\}$. Therefore t is $\lambda-$bounded iff $\Lambda(t) \in \ell_\infty$. $p-$absolute convergence of type λ and of sequence t and $\Lambda(t) \in \ell_\infty$ both are equivalent. The matrix Λ is a lower triangular matrix.
Recently, $c_0^\lambda(\Delta)$ and $c^\lambda(\Delta)$ have been defined by (M. Mursaleen, 2010) and shown the inclusion relation $c_0 \subset c_0^\lambda(\Delta) \subset c^\lambda(\Delta), c \subset c^\lambda(\Delta)$. Finally, we define the the sequence $s(\lambda) = \{s_k(\lambda)\}$ for the use of $\Lambda-$transform of a sequence t that is $s(\lambda) = \Lambda(t)$ and so,

$$s_k(\lambda) = \sum_{m=0}^{k}\left(\frac{\lambda_m - \lambda_{m-1}}{\lambda_k}\right)t_m\ ;$$

III. The sequence spaces $\boldsymbol{\ell_p^\lambda(\Delta)}$ and $\boldsymbol{\ell_\infty^\lambda(\Delta)}$ of non-absolute type

In this section we introduce the sequence spaces $\ell_p^\lambda(\Delta)$ and $\ell_\infty^\lambda(\Delta)$ as follows,

$$\ell_p^\lambda(\Delta) = \left\{t = (t_k) \in w : \sum_{n=0}^{\infty}\left|\frac{1}{\lambda_n}\sum_{k=0}^{n}\Delta\lambda_k\,\Delta x_k\right|^p \in \mathbb{R}\right\};\ (0 < p < \infty)$$

and

$$\ell_\infty^\lambda(\Delta) = \left\{t = (t_k) \in w : \sup_n\left|\frac{1}{\lambda_n}\sum_{k=0}^{n}\Delta\lambda_k\,\Delta x_k\right| \in \mathbb{R}\right\}$$

Where, $\Delta\lambda_k = \lambda_k - \lambda_{k-1}$, $\Delta x_k = x_k - x_{k-1}$. Obviously, $\ell_\infty^\lambda(\Delta)$ and $\ell_p^\lambda(\Delta), (0 < p < \infty)$ are sequence spaces.

Theorem 3.1. We have the following:

(a) For, $p \in (0,1)$, then $\ell_p^\lambda(\Delta)$ is a complete p-normed space with the p-norm $\| x \|_{\ell_p^\lambda} = \| \Lambda(x) \|_{\ell_p}$, i.e.

$$\| x \|_{\ell_p^\lambda} = \sum_n |\Lambda_n(x)|^p;\ (0 < p < 1)$$

(b) If $1 \le p \le \infty$, then ℓ_p^λ is a BK-space with the norm $\| x \|_{\ell_p^\lambda} = \| \Lambda(x) \|_{\ell_p}$, that is

$$\| x \|_{\ell_p^\lambda} = \left(\sum_n |\Lambda_n(x)|^p\right)^{1/p};\ (1 \le p < \infty)$$

and

$$\| x \|_{\ell_\infty^\lambda} = \sup_n |\Lambda_n(x)|$$

Theorem 3.2. The sequence space $\ell_p^\lambda(\Delta)$ of non-absolute type is isometrically isomorphic to the space $\ell_p(\Delta)$ for $p > 0$.

Proof. First we show the existence of an isometric isomorphism between the spaces $\ell_p^\lambda(\Delta)$ and $\ell_p(\Delta)$. For, let $p > 0$ and consider the transformation T defined, from $\ell_p^\lambda(\Delta)$ to ℓ_p by $x \longmapsto y(\lambda) = Tx$. Then, we have $Tx = y(\lambda) = \Lambda(x) \in \ell_p$ for every $x \in \ell_p^\lambda$.Also, the linearity of T is trivial. Further, it is easy to see that $x = 0$ whenever $Tx = 0$ and hence T is injective.

Furthermore, let $y = (y_k) \in \ell_p$ be given and define the sequence $x = \{x_k(\lambda)\}$ by

$$x_k(\lambda) = \sum_{j=k-1}^{k} (-1)^{k-j} \frac{\lambda_j}{\lambda_k - \lambda_{k-1}} y_j;\ (k \in \mathbb{N})$$

Then,

$$\Lambda_n(x) = \frac{1}{\lambda_n}\sum_{k=0}^{n} (\lambda_k - \lambda_{k-1}) x_k(\lambda)$$
$$= \frac{1}{\lambda_n}\sum_{k=0}^{n} \sum_{j=k-1}^{k} (-1)^{k-j}\lambda_j y_j$$
$$= \frac{1}{\lambda_n}\sum_{k=0}^{n} (\lambda_k y_k - \lambda_{k-1} y_{k-1})$$
$$= y_n$$

This shows that $\Lambda(x) = y$ and since $y \in \ell_p$, we obtain that $\Lambda(x) \in \ell_p$. Thus, we deduce that $x \in \ell_p^\lambda$ and $Tx = y$. Hence T is surjective.

Moreover, for any $x \in \ell_p^\lambda(\Delta)$, we have by Theorem 3.1 that

$$\| Tx \|_{\ell_p} = \| y(\lambda) \|_{\ell_p} = \| \Lambda(x) \|_{\ell_p} = \| x \|_{\ell_p^\lambda}$$

which shows that T is p-norm and norm preserving in the cases of $0 < p < 1$ and $1 \le p \le \infty$, respectively. Hence T is isometry. Consequently, the spaces $\ell_p^\lambda(\Delta)$ and $\ell_p(\Delta)$ are isometrically isomorphic for $0 < p \le \infty$. This concludes the proof.

IV. Some inclusion relations

We show that the inclusion $\ell_\infty \subset \ell_\infty^\lambda(\Delta)$ holds and characterize the case in which the inclusion $\ell_p \subset \ell_p^\lambda(\Delta)$ holds for $p > 1$.

Lemma 4.1. For any sequence $t = (t_k) \in w$, the equalities

$$S_n(t) = t_n - \Lambda_n(t);\ (n \in \mathbb{N})$$

and

$$S_n(t) = \frac{\lambda_{n-1}}{\lambda_n - \lambda_{n-1}}[\Lambda_n(t) - \Lambda_{n-1}(t)];\ (n \in \mathbb{N})$$

hold, where $S(t) = \{S_n(t)\}$ is the sequence defined by

$$S_0(t) = 0 \text{ and } S_n(t) = \frac{1}{\lambda_n}\sum_{k=1}^{n} \lambda_{k-1}(t_k - t_{k-1});\ (n \ge 1)$$

Lemma 4.2. For any sequence $\lambda = (\lambda_k)_{k=0}^\infty$, we have

(a) $\left(\frac{\lambda_k}{\lambda_k - \lambda_{k-1}}\right)_{k=0}^\infty \notin \ell_\infty$ if and only if $\lim\inf_{k\to\infty} \frac{\lambda_{k+1}}{\lambda_k} = 1$.

(b) $\left(\frac{\lambda_k}{\lambda_k - \lambda_{k-1}}\right)_{k=0}^\infty \in \ell_\infty$ if and only if $\lim\inf_{k\to\infty} \frac{\lambda_{k+1}}{\lambda_k} > 1$.

Theorem 4.3. If $0 < p < q < \infty$, then the inclusion $\ell_p^\lambda(\Delta) \subset \ell_q^\lambda(\Delta)$ strictly holds.

Proof. Let $0 < p < q < \infty$. Then, it follows by the inclusion $\ell_p \subset \ell_q$ that the inclusion $\ell_p^\lambda(\Delta) \subset \ell_q^\lambda(\Delta)$ holds. Further, since the inclusion $\ell_p \subset \ell_q$ is strict, there is a sequence $t = (t_k)$ in ℓ_q but not in ℓ_p, i.e., $t \in \ell_q \setminus \ell_p$. Let us now define the sequence $s = (s_k)$ in terms of the sequence t as follows:

$$s_k = \frac{\lambda_k t_k - \lambda_{k-1} t_{k-1}}{\lambda_k - \lambda_{k-1}};\ (k \in \mathbb{N})$$

Then, we have for every $n \in \mathbb{N}$ that

$$\Lambda_n(s) = \frac{1}{\lambda_n}\sum_{k=0}^{n} (\lambda_k t - \lambda_{k-1} t_{k-1}) = t_n$$

which shows that $\Lambda(s) = t$ and hence $\Lambda(s) \in \ell_q \setminus \ell_p$. Thus, the sequence s is in ℓ_q^λ but not in ℓ_p^λ. Hence, the inclusion $\ell_p^\lambda \subset \ell_q^\lambda$ is strict. This concludes the proof.

Theorem 4.3. The inclusions $\ell_p^\lambda(\Delta) \subset c_0^\lambda(\Delta) \subset c^\lambda(\Delta) \subset \ell_\infty^\lambda(\Delta)$ strictly hold, where $p > 0$.

Proof. Since the inclusion $c_0^\lambda(\Delta) \subset c^\lambda(\Delta)$ strictly holds, it is enough to show that the inclusions $\ell_p^\lambda(\Delta) \subset c_0^\lambda(\Delta)$ and $c^\lambda(\Delta) \subset \ell_\infty^\lambda(\Delta)$ are strict, where $p > 0$.

Firstly, it is trivial that the inclusion $\ell_p^\lambda(\Delta) \subset c_0^\lambda(\Delta)$ holds for $p > 0$, since $t \in \ell_p^\lambda(\Delta)$ implies $\Lambda(t) \in \ell_p$ and hence $\Lambda(t) \in c_0$ which means that $t \in c_0^\lambda(\Delta)$. Further, to show that this inclusion is strict, let $p > 0$ and consider the sequence $t = (t_k)$ defined by

$$t_k = \frac{1}{(k+1)^{1/p}};\ (k \in \mathbb{N})$$

Then $t \in c_0$ and hence $t \in c_0^\lambda(\Delta)$, since the inclusion $c_0 \subset c_0^\lambda(\Delta)$ holds. On the other hand, we have for every $n \in \mathbb{N}$ that

$$|\Lambda_n(t)| = \frac{1}{\lambda_n}\sum_{k=0}^{n} \frac{\lambda_k - \lambda_{k-1}}{(k+1)^{1/p}}$$

$$\geq \frac{1}{\lambda_n(n+1)^{1/p}}\sum_{k=0}^{n} (\lambda_k - \lambda_k - 1)$$

$$= \frac{1}{(n+1)^{1/p}}$$

which shows that $\Lambda(t) \notin \ell_p$ and hence $x \notin \ell_p^\lambda(\Delta)$. Thus, the sequence x is in $c_0^\lambda(\Delta)$ but not in $\ell_p^\lambda(\Delta)$. Therefore, the inclusion $\ell_p^\lambda(\Delta)\ \subset c_0^\lambda(\Delta)$ is strict for $0 < p < \infty$.

Similarly, it is also clear that the inclusion $c^\lambda(\Delta) \subset \ell_\infty^\lambda(\Delta)$ holds. To show that this inclusion is strict, we define the sequence $s = (s_k)$ by

$$s_k = (-1)^k \left(\frac{\lambda_k + \lambda_{k-1}}{\lambda_k - \lambda_{k-1}}\right);\ (k \in \mathbb{N})$$

Then, we have for every $n \in \mathbb{N}$ that

$$\Lambda_n(s) = \frac{1}{\lambda_n}\sum_{k=0}^{n} (-1)^k(\lambda_k + \lambda_{k-1}) = (-1)^n$$

which shows that $\Lambda(s) \in \ell_\infty \setminus c$. Thus, the sequence s is in $\ell_\infty^\lambda(\Delta)$ but not in $c^\lambda(\Delta)$ and hence $c^\lambda \subset \ell_\infty^\lambda(\Delta)$ is a strict inclusion.

Theorem 4.4. The relation $\ell_\infty \subset \ell_\infty^\lambda(\Delta)$ holds. Further, the equality holds if and only if $S(t) \in \ell_\infty$ for every sequence $x \in \ell_\infty^\lambda(\Delta)$.

Proof. The first part of the theorem is immediately obtained from Lemma 2.1, and so we turn to the second part. For, suppose firstly that the equality $\ell_\infty^\lambda(\Delta) = \ell_\infty$ holds. Then, the inclusion $\ell_\infty^\lambda(\Delta) \subset \ell_\infty$ holds and $S(t) \in \ell_\infty$ for every $t \in \ell_\infty^\lambda$.

Conversely, suppose that $S(t) \in \ell_\infty$ for every $t \in \ell_\infty^\lambda(\Delta)$. Then, the inclusion $\ell_\infty^\lambda(\Delta) \subset \ell_\infty$ holds. Combining this with the inclusion $\ell_\infty \subset \ell_\infty^\lambda(\Delta)$, we get the equality $\ell_\infty^\lambda(\Delta) = \ell_\infty$. This completes the proof.

V. The basis for the space $\ell_p^\lambda(\Delta)$

We discuss about the basis of $\ell_p^\lambda(\Delta)$. If a normed space X contains a sequence (b_n) with the property that for every $t \in X$ there is a unique sequence (α_n) of scalars such that

$$\lim_{n\to\infty} \|t - (\alpha_0 b_0 + \alpha_1 b_1 + \cdots + \alpha_n b_n)\| = 0$$

then (b_n) is called a Schauder basis (or briefly basis) for X. The series $\sum_k \alpha_k b_k$ which has the sum t is then called the expansion of x with respect to (b_n), and written as $t = \sum_k \alpha_k b_k$.

Theorem 5.1. Let $1 \leq p < \infty$ and define the sequence $e_\lambda^{(k)} \in \ell_p^\lambda(\Delta)$ for every fixed $k \in \mathbb{N}$ by

$$\left(e_\lambda^{(k)}\right)_n = \begin{cases} (-1)^{n-k}\dfrac{\lambda_k}{\lambda_n - \lambda_{n-1}}; & (k \leq n \leq k+1) \\ 0; & (\text{ otherwise }) \end{cases} (n \in \mathbb{N})$$

Then, the sequence $\left(e_\lambda^{(k)}\right)_{k=0}^{\infty}$ is a basis for the space $\ell_p^\lambda(\Delta)$ and every $t \in \ell_p^\lambda(\Delta)$ has a unique representation of the form

$$t = \sum_k \Lambda_k(t) e_\lambda^{(k)}$$

Proof. Let $1 \leq p < \infty$. Then, $\Lambda\left(e_\lambda^{(k)}\right) = e^{(k)} \in \ell_p (k \in \mathbb{N})$ and hence $e_\lambda^{(k)} \in \ell_p^\lambda(\Delta)$ for all $k \in \mathbb{N}$.

Further, let $t \in \ell_p^\lambda(\Delta)$ be given. For every non-negative integer m, we put

$$t^{(m)} = \sum_{k=0}^{m} \Lambda_k(t) e_\lambda^{(k)}$$

Then, we have that

$$\Lambda\left(t^{(m)}\right) = \sum_{k=0}^{m} \Lambda_k(t)\Lambda\left(e_\lambda^{(k)}\right) = \sum_{k=0}^{m} \Lambda_k(t) e^{(k)}$$

and hence

$$\Lambda_n\left(t - t^{(m)}\right) = \begin{cases} 0; & (0 \leq n \leq m) \\ \Lambda_n(t); & (n > m) \end{cases} (n, m \in \mathbb{N})$$

Now, for any given $\epsilon > 0$ there is a non-negative integer m_0 such that

$$\sum_{n=m_0+1}^{\infty} |\Lambda_n(t)|^p \leq \left(\frac{\epsilon}{2}\right)^p$$

Therefore, we have for every $m \geq m_0$ that

$$\|x - x^{(m)}\|_{\ell_p^\lambda} = \left(\sum_{n=m+1}^{\infty} |\Lambda_n(t)|^p\right)^{1/p}$$
$$\leq \left(\sum_{n=m_0+1}^{\infty} |\Lambda_n(t)|^p\right)^{1/p}$$
$$\leq \frac{\epsilon}{2} < \epsilon$$

which shows that $\lim_{m\to\infty} \|t - t^{(m)}\|_{\ell_p^\lambda} = 0$.

Finally, of $t \in \ell_p^\lambda(\Delta)$. For this, suppose that $t = \sum_k \alpha_k(x) e_\lambda^{(k)}$. Since the linear transformation T defined from $\ell_p^\lambda(\Delta)$ to ℓ_p, in the proof of Theorem 3.3, is continuous, we have

$$\Lambda_n(t) = \sum_k \alpha_k(t)\Lambda_n\left(e_\lambda^{(k)}\right) = \sum_k \alpha_k(t)\delta_{nk} = \alpha_n(t);\ (n \in \mathbb{N})$$

Hence, $t \in \ell_p^\lambda(\Delta)$ is unique. This completes the proof.

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