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An EOQ Model for Deteriorating Items with Return Policy having Time and Selling Price Dependent Demand with Shortages

[1]Ranjana Gupta, [2]Meenakshi Srivastava
[1]Assistant Professor, [2]Professor
Department of Statistics, Institute of Social Sciences
Dr. Bhimrao Ambedkar University, Agra-282004 (U.P.) INDIA.

ABSTRACT

The paper deals with the construction of EOQ model where demand is dependent upon time and selling price. The demand is taken to be constant for certain period of time and then it is a function of time and selling price. To make our model more realistic, return policy with shortages of the product is incorporated, with the addition that the product can be returned and resold by the retailer. It is assumed that the retailer will reimburse half of the price of the returned product to the customer. The primary purpose of this inventory model is to determine the optimum time, optimum order quantity and optimum retailer's profit earned per unit time. A numerical example is also presented and sensitivity analysis is carried out to highlight the findings of the suggested inventory model.

Keywords: deteriorating items, time and selling price demand, shortages, return policy

1. INTRODUCTION

Inventory management and control system deals with demand and supply chain problem. The base of any business is the demand and supply of goods. As it is seen, that a large quantity of goods in store will lead the customer to buy more goods which creates the greater demand of the goods. In the competitive market situation, several companies confer various types of incentives and facilities during sale of the product to their customers with certain terms and conditions. The return policies are often offered by the sellers to their customers who are unsatisfied with their purchase. They return the product and get full or partial refund either in terms of money or some gift vouchers as stated by the policies of the company. In order to increase the sale, most of the companies offer return polices to their customers.

Depending upon the return policies, the customers who are unsatisfied with the qualities of the products are returning their products to the companies/retailers during past years. Very few literatures is available on the development of inventory models for deteriorating items with return policies incorporating variation in demand and selling price together with shortages. Keeping this into account, the present study focuses its attention on the development of such type of model by including these features. Deterioration of any item is an important factor in the inventory management. The age of the inventory has a negative impact on demand due to loss of consumer's confidence on the quality of such products and physical loss of the materials. Ghare and Scharader [9] were initiate the concept of deterioration and developed an inventory model with constant deterioration rate. In the last few decades and also in most of the present models deterioration in items is considered and inventory cost includes the deterioration cost. Inventory models on deteriorating items with the return policy have been developed by many researchers. Vlachos and Dekker[7] and Mostand et. al. [6] proposed an inventory model for deteriorating items with resalable returns with the following assumptions: (i) returns are allowed only within a certain time frame after that no return is accepted (ii) money is fully reimbursed by retailer (iii) returns can be resold and returned several times per period. An EOQ model with resalable returns for deteriorating items was developed by Wang et. al.[5]. Ghoreishi et. al.[4] suggested an EOQ model under inflation to determine the pricing and ordering strategy of deteriorating items under customer returns.

Khouja et. al. [2] proposed retailer's performance under the effect of price adjustment polices and returns. Kumari and Kanti De [1] have been proposed an inventory model with jointly analysing the impact of both trade credit and return policy on deteriorating items with resalable returns.

In classical inventory models, demand rate is assumed to be constant for the whole cycle. Sometimes demand may be consider as function of time, selling price of the goods and stock in the inventory. Selling price of the product is very important factor in any inventory system especially in case of deteriorating products. To increase the sale of the product or increase the demand, various inventory models have been developed with selling price dependent demand rate. Polatoglu and Sahin[8] developed a periodic inventory model with stochastic demand which is dependent on the unit price. Saha and Sen [3] developed an inventory model for deteriorating items having time and price dependent demand with shortages under the effect of inflation.

In the present paper, an economic order quantity model is developed with certain features like the demand is constant for certain period and after that period it is jointly dependent on time and selling price of the product. The logic behind for taking such type of demand is that when a product is launched in the market then at the beginning of time demand is stable that is not increasing or decreasing; it will be constant for a short period of time and after that certain period demand increases or decreases according to the choice of the customers. In the present paper, demand is considered to be jointly dependent on time and selling price of the product after certain period. Deterioration of the product is also included in the period where demand is varying with time and selling price. To increase the sale, retailer offers a return policy to its customers with certain conditions. Customers are allowed to return the product after a certain period of time in the replenishment cycle. Product returned can be resold at the same selling price but the retailer returns half of the price for the returned goods to the customers. It is also assumed that the return rate is jointly dependent on demand function.

The aim of the present paper is to determine the optimum cycle time, optimum order quantity and optimum retailer's profit earned by unit time, under the above assumptions.

This paper is structured as follows:

Section 2, provides the notations and assumptions of the model. The mathematical development of the suggested model is presented in section 3. Its applications are done in section 4 with the help of a numerical example to highlight the findings. Sensitivity analysis with interpretations is done in section 5 to demonstrate the results while section 6 ends up with some valuable conclusions and suggestions.

2. ASSUMPTIONS AND NOTATIONS

Assumptions:

i. A single item is considered over a prescribed period of T unit of time.
ii. Replenishment rate is infinite with delivery lead time is zero.
iii. Shortages are allowed.

iv. The time horizon of the inventory system is infinite. Only a typical planning schedule of length T is considered, all remaining cycles are identical.

v. Demand is constant at the rate of 'a' units per unit of time for the time period $(0, \mu)$. For the time period (μ, t_1), demand rate is a function of time and selling price of the product P, which is defined as,

$$D(P,t) = (a + b\,t)\,P^{-\alpha}, \quad \text{where } a > 0,\ b > 0 \text{ and } \alpha > 0$$

vi. There is no repair or replacement of deteriorated units.

vii. It is assumed that customers are allowed to return the product during the period (μ, t_1). So, the customers return increase with the goods sold.

$$R(P, t) = \beta\, D(P,t), \qquad \text{where } 0 < \beta < 1$$

viii. The retailers can resold the returned product at the same selling price and reimburse half of the price of the returned product to the customers.

ix. The holding cost, shortage cost, unit cost, deterioration cost remain constant for the whole cycle.

x. There is no deterioration cost for the period $(0, \mu)$ when the demand rate is constant. The deterioration rate is constant say 'θ' which is very small for the period (μ, t_1), where demand is varying with time and selling price.

Notations:

$I(t)$: On hand inventory level at time t

t_1: Time period at which inventory level reaches to zero

Q: number of items received at the beginning of the period

C: Unit cost/unit

P: Selling price of the product/unit

C_1: Inventory holding cost per unit per unit of time

C_2: Shortage cost per unit per unit of time

C_3: Set-up cost per cycle

μ: The time period at which demand varies with time and selling price and also deterioration of the product starts

θ: Constant deterioration rate; $0 < \theta < 1$

TP: Total profit per unit time

T: Replenishment cycle time

T^*: Optimum replenishment cycle time

Q^*: Optimum order quantity

TP^*: Optimum total profit per unit time

3. MATHEMATICAL FORMULAION OF MODEL

According to the assumption and notation discussed in the previous section, the behaviour of inventory system at time 't' can be depicted in the following inventory time diagram (fig. 1).

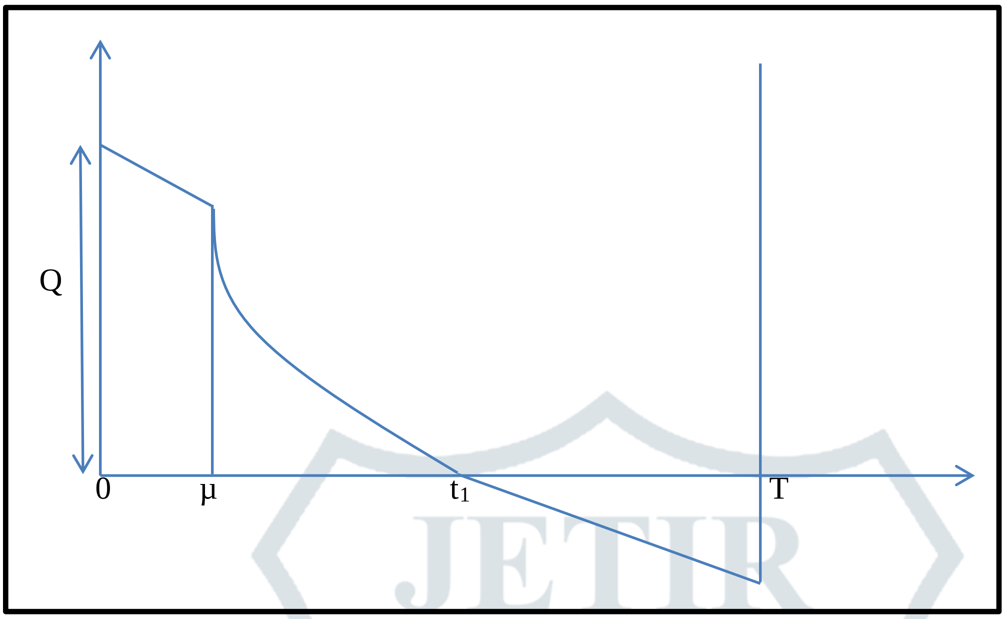

Figure 1. INVENTORY-TIME DIAGRAM

From figure 1, it is seen that initially inventory has 'Q' units of goods. During the time interval (0, μ), the inventory level decreases due to constant demand say 'a' units per unit of time. During this period, deterioration is not considered. After time t = μ, deterioration of the product takes place at the constant rate 'θ'. During the time period (μ, t_1), inventory level reduces due to the combined effect of varying demand, product return and deterioration. At time t = t_1, inventory level falls to zero. After the time t = t_1, shortages occurred and accumulate to the level S at t = T. Thereafter, at t = T, a replenishment order of Q units is placed which marks the beginning of the next cycle.

Retailer also permits its customers to return products during the cycle (μ, t_1) with the condition that the retailer will reimburse half of the amount of the product to the customer. The main aim is to find out the optimum time t_1^*, that maximizes the total profit earned by the retailer.

In time interval (0, μ) demand is constant say 'a' units per unit of time.

Therefore, total demand in time interval (0, μ) is = a μ

Thus, the inventory level is reduced by the factor 'a μ' and (Q - a μ) level of inventory is remaining for the time period (μ, t_1).

The status of inventory level I (t) over the period (μ, T) is given by first order differential equations.

$$\frac{dI(t)}{dt} + \theta I(t) = -(a+bt)P^{-\alpha} + \beta (a+bt)P^{-\alpha} \qquad \mu < t < t_1 \qquad (1)$$

$$\frac{dI(t)}{dt} = -(a+bt)P^{-\alpha} \qquad t_1 < t < T \qquad (2)$$

Solution of the differential equation (1) with the boundary conditions at t = t_1, I(t) = 0 becomes

$$I(t) = \frac{(\beta-1)P^{-\alpha}}{\theta^2}\{[(a\theta-b)+b\theta t]-[(a\theta-b)+b\theta t_1]\, e^{\theta(t_1-t)}\}$$

Solution of the differential equation (2) with the boundary conditions at $t = t_1$, $I(t) = 0$ becomes

$$I(t) = P^{-\alpha}[a(t_1-t)+\frac{b(t_1^2-t^2)}{2}]$$

Holding cost for the time period $(0, \mu)$ is = $C_1\mu Q-\frac{1}{2}C_1 a\mu^2$

Holding cost for the time period (μ, t_1) is = $C_1\frac{(\beta-1)P^{-\alpha}}{\theta^2}\left[-\frac{b\theta t_1^2}{2}-\frac{b\theta\mu^2}{2}+b\theta\mu t_1\right]$

Deterioration cost = $C\theta\int_{\mu}^{t_1}\frac{(\beta-1)P^{-\alpha}}{\theta^2}\left[\left\{[(a\theta-b)+b\theta t]-[(a\theta-b)+b\theta t_1]\, e^{\theta(t_1-t)}\right\}\right]dt$

$$= C\theta\frac{(\beta-1)P^{-\alpha}}{\theta^2}\left[-\frac{b\theta t_1^2}{2}-\frac{b\theta\mu^2}{2}+b\theta\mu t_1\right]$$

Shortage cost = $C_2\int_{t_1}^{T}P^{-\alpha}[a(t_1-t)+\frac{b(t_1^2-t^2)}{2}]\,dt$

$$=P^{-\alpha}C_2\left[a t_1(T-t_1)-a\frac{(T^2-t_1^2)}{2}+b t_1^2\frac{(T-t_1)}{2}-b\frac{(T^3-t_1^3)}{6}\right]$$

Total cost per unit time = (Holding cost + Deterioration cost + Shortage cost +Purchase cost)/T

For the sake of simplicity, use the substitution $t_1 = \lambda T$

$$TC=T^3\left[\frac{P^{-\alpha}C_2 b\lambda(1-\lambda)}{2}-\frac{P^{-\alpha}C_2 b(1-\lambda^3)}{6}\right]$$
$$+T^2\left[-bAD\theta^2\lambda^2-\frac{C_1 Ab\theta\lambda^2}{2}-\frac{CAb\theta^2\lambda^2}{2}+C_2 aP^{-\alpha}\lambda(1-\lambda)-\frac{C_2 aP^{-\alpha}(1-\lambda^2)}{2}\right]$$
$$+T\left[-DA(B+b)\theta\lambda+C_1 Ab\theta\lambda\mu+CAb\theta^2\lambda\mu\right]$$
$$+\left[DA\theta\mu(B+b)+Da\mu-\frac{aC_1\mu^2}{2}-\frac{C_1 Ab\theta\mu^2}{2}-\frac{CAb\theta^2 b\mu^2}{2}+C_3\right]$$

Where, $A=\frac{(\beta-1)P^{-\alpha}}{\theta^2}$,

$B=(a\theta-b)$

$D=C_1\mu+C$

$$TC = XT^3 + YT^2 + ZT + R$$

where,

$$X = \frac{P^{-\alpha} C_2 b\lambda(1-\lambda)}{2} - \frac{P^{-\alpha} C_2 b(1-\lambda^3)}{6}$$

$$Y = -bAD\theta^2\lambda^2 - \frac{C_1 Ab\theta\lambda^2}{2} - \frac{CAb\theta^2\lambda^2}{2} + C_2 a P^{-\alpha}\lambda(1-\lambda) - \frac{C_2 a P^{-\alpha}(1-\lambda^2)}{2}$$

$$Z = -DA(B+b)\theta\lambda + C_1 Ab\theta\lambda\mu + CAb\theta^2\lambda\mu$$

$$R = DA\theta\mu(B+b) + Da\mu - \frac{aC_1\mu^2}{2} - \frac{C_1 Ab\theta\mu^2}{2} - \frac{CAb\theta^2 b\mu^2}{2} + C_3$$

Total Sales Revenue = $P\{a\mu + \int_{\mu}^{t_1}(a+bt)P^{-\alpha}\,dt - \frac{\beta}{2}\int_{\mu}^{t_1}(a+bt)P^{-\alpha}\,dt\}$

$$TSR = T^2\left[\frac{Mb\lambda^2}{2}\right] + T\left[Ma\lambda\right] + \left[Pa\mu - a\mu M - \frac{Mb\mu^2}{2}\right]$$

Where,

$$M = P^{1-\alpha}\left(1 - \beta/2\right)$$

$$TSR = UT^2 + VT + W$$

where,

$$U = \frac{Mb\lambda^2}{2} - PbA\theta^2\lambda^2$$

$$V = Ma\lambda - Pa\lambda(B+b)\theta$$

$$W = -a\mu M - \frac{Mb\mu^2}{2} + PA\theta\mu(B+b)$$

Total Profit Per Unit Time = [Total Sale Revenue – Total Cost] / T

$$TP/\text{unit time} = \frac{1}{T}\{[UT^2 + VT + W] - [XT^3 + YT^2 + ZT + R]\}$$

$$= \frac{1}{T}[-XT^3 + (U-Y)T^2 + (V-Z)T + (W-R)] \quad \text{.................(3)}$$

$$Q = a\mu + A[-b\theta^2\lambda^2T^2 - \lambda T\theta(B+b) + \theta\mu(B+b)] \quad \text{........................(4)}$$

To maximize the profit function, differentiate the profit function w.r.t 'T' and equate it to zero to get the optimum cycle time.

$$\frac{\partial TP(T)}{T} = 2XT^3 - (U-Y)T^2 + (W-R) = 0 \quad \text{....................(5)}$$

Equation (5) can be solved for a positive $T=T^*$. T^* will be an optimal solution, provided the following condition is satisfied for $T=T^*$.

$$\frac{\partial^2 TP(T)}{\partial T^2} < 0$$

After getting the optimum value of T*, optimum Q =Q* from equation (4) and optimum total profit TP* from equation (3) also calculated.

4. NUMERICAL ILLUSTRATION

To illustrate the results obtained for the suggested model, a numerical example with the following parameter values in considered;

$$a = 200,\ b = 4,\ \alpha = 0.8,\ \beta = 0.1,\ \theta = 0.08,\ C = Rs.\ 20/unit,\ C_1 = Rs.2/unit,$$
$$C_2 = Rs.2/unit,\ C_3 = Rs.100/order,\ P = 30/unit, \mu = 1 day,\ \lambda = 0.6$$

From the above parameter values the optimum policies of the system are as given below:

T* = 31.4 Days, Q* = 518 Units, TP* = Rs.144.47

5. SENSITIVITY ANALYSIS AND INTEPRETATIONS

A sensitivity analysis is done to know the effect of changes in the system parameters by increasing or decreasing the parameters by 5% to 10%.

Table 1

Effect of change of parameters on optimum cycle time, optimum order quantity and optimum total profit

Changing parameters	% change	T*	Q*	TP*
α	-10%	28	564	172.58
	-5%	30	545	151.07
	+5%	32	485	143.85
	+10%	34	468	122.83
β	-10%	31.6	524	143.75
	-5%	31.4	520	144.20
	+5%	28.6	483	153.19
	+10%	28	474	154.38

λ	-10%	25	414	169.50
	-5%	27	450	160.57
	+5%	37	542	124.71
	+10%	46	789	99.78
θ	-10%	29.4	494	148.56
	-5%	29	489	150.20
	+5%	28	447	152.99
	+10%	28	439	154.26

INTERPRETATIONS

From the above sensitivity analyses following interpretations are made:

(i) As the demand parameter ‘α’ decreases up to 10%, then the total profit is increasing up to 20% approximately. The inventory cycle time reduces and optimum order quantity increases with the reduction in parameter ‘α’

(ii) As the parameter ‘β’ which is the fraction of demand returned by the costumers increases up to 10%, then the then the total system profit is increasing up to only 6%. The cycle time decreases and order quantity also reduced on the increment of the value of parameter ‘β’.

(iii) As the parameter ‘λ’ which is the fraction of cycle time in which the inventory reaches to zero and from where shortages start, decreases up to 10%, then the total profit increases up to 17% approximately. The inventory cycle time and optimum order quantity both decreases with the reduction in parameter ‘λ’.

(iv) As the deteriorating parameter ‘θ’ increases up to 10%, then total profit increases up to 6% approximately. The optimum cycle time and optimum order quantity both reduces simultaneously in the increment of parameter ‘θ’.

6. CONLUSION

An Economic Order Quantity model for deteriorating items with a single item is developed for single replenishment cycle with return policy and shortages. Some of the researcher have developed inventory model with resalable returns for perishable products and the recent research recommends following a restrictive return policy. Return policy is beneficial for the business as it attracts customers to buy more. In this research paper return policy is implemented for the customer with half of the selling price is reimbursed for the returned product. The returned product can be resold at the same selling price. This resalable and return scheme is beneficial not only for the seller but for the customers also.

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