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# AN ANALYTICAL FRAMEWORK WITH A BUDGET CONSTRAINT FOR THE MULTI-ITEM ECONOMIC ORDER QUANTITY MODEL WITH ALLOWABLE PAYMENT DELAYS

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***Abstract :*** In earlier research, Purchase cards and inventory control: an analytical framework was conducted by A. Chandrashekar and M. Gopalakrishnan. Prod. Plan. & Cont., 2005, 16(5), 437–443), we created a framework to contrast ordering goods directly from a supplier using "purchase cards" (P-cards) with getting direct credit from the provider. In the instance of a single item, we demonstrated that there's a critical ordering cost, below which the P-card can be utilised only in the event that the actual price of placing an order using the P-card is below this threshold amount. This paper expands on the methodology that we previously developed to determine the ideal replenishment amount for multi-item ordering when dealing with budgetary constraints, allowable payment delays, and partial payments that incur penalties. Even in cases where the interest generated exceeds the interest charged, the research suggests that it is more cost-effective for the buyer to complete all of the payments.
This note more faithfully captures the truth.

***Keywords:*** **partial payment; budget limit; supplier credit; multi-item EOQ.**

## I. INTRODUCTION

The issue with multi-item, single-echelon inventory management has been thoroughly studied. Managing several things while adhering to real-world restrictions, such the maximum amount that can be spent on inventory, can be a very challenging task. We built an analytical approach for the single-item situation with allowable payment delays and budget constraints in a previous publication (Chandrashekar and Gopalakrishnan 2005). In the instance of a single item, we demonstrated the existence of a critical ordering cost, which limits the use of the purchase card (P-card) to situations in which the real cost of placing an order with the P-card is less than this critical value. We expand the paradigm in this note to include multi-item scenarios with allowable partial payments. Furthermore, this expansion more truly depicts the ordering process as it actually is.

## II. MODEL FORMULATION

Our notation for describing the variables for decisions and the parameters is as follows.

$n$ Number of products.
$D_i$ Annual demand for product i.
$S_i$ Ordering cost per order for product i.
$c_i$ Unit purchasing cost for product i.
$p_i$ Unit selling price for product i.
$h_i$ Fraction holding cost per unit per year, excluding interest charges.
$\delta$ Interest charged per dollar per year.
$\gamma$ Interest earned per dollar per year.
$\alpha$ Percentage of the total order paid in the current cycle.

T Replenishment time interval.
$t_i$ Time duration between placing the orders and receiving the orders.
$q_i$ Time duration between placing the orders and making the payment (total credit duration or permissible delay).
B Buyer's budget limit.
$Q_i$ Order quantity (a decision variable) for product i.
Z(Q) Average total annual cost (TAC) that includes the order cost and inventory cost for all products.
$Q_i^*$ Optimal order quantity

**III. ASSUMPTIONS**

1. We make the assumption that there will always be a demand for every product and that shortages are not permitted.
2. Partial payments are also accepted. The client has the option to pay only a portion of the total and postpone paying interest on the balance until the following payment month.
3. The supplier consents to a postponed payment. If the buyer pays within a predetermined window of time after receiving the products, there are no extra fees. We call this predetermined window of time the supplier credit or allowable delay. Since q > t, the delay exceeds the replenishment interval.
Payment is made upon receipt of the products (q > t), provided that the buyer's budget allows for both the current transaction's entire cost of purchase and the remaining balance from the prior period's payment. If the entire amount is paid, the supplier permits a delay time (q - t) during which no interest is assessed.
(i) The price of placing a product order I. This is determined by the order quantity Qi and the demand Di.

$$\frac{Di}{Qi} S_i$$

(ii) The price of insurance, taxes, storage, and other expenses; the cost of capital is not included. Since this expense is incurred during the entire year, it is provided by

$$\frac{Qi}{2} h_i c_i$$

(iii) Interest accrued on sales during the allowable postponement. The sales income for the duration of the allowed delay,

$$\frac{Di}{2} (q_i - t_i) p_i,$$

accrues interest at the rate of · per dollar annually for the duration of the allowable delay. Interest income, thus, for every cycle equals

$$\$ \frac{Di}{2} p_i \gamma (q_i - t_i)^2$$

and the year's total interest income is

$$D_i p_i \gamma \left[(qi - ti) - \frac{T}{2}\right]$$

(iv) Interest applied to the outstanding amount. Interest is applied to the outstanding sum ($\$(1-\alpha) c_i Q_i$ ) until the following payment period. This leads to an extra yearly expense of

$$\$(1-\alpha) c_i Q_i \delta T \frac{Di}{Qi},$$

which simplifies to $(1-\alpha)c_i Q_i \delta$
(v) Interest earned on outstanding debt. Up to the subsequent payment period, the outstanding amount ($\$(1-\alpha) c_i Q_i$ ) may accrue interest at the rate of $\gamma$. As a result, $\$(1-\alpha) c_i Q_i \gamma$ is the annual income.
The sum of the five cost components shown above for all items may now be used to indicate the overall annual inventory cost across all products. Therefore, the current challenge is to reduce the overall cost of inventory while staying within the allocated budget.
Minimise

$$Z(Q) = \sum_{i=1}^{n} \left( \frac{DiSi}{Qi} + \frac{Qihici}{2} + (1-\alpha)ci\delta Qi - Di\left[(qi - ti) - \frac{T}{2}\right] pi\gamma - (1-\alpha)ci\gamma Qi \right) \quad (1)$$

Subject to :

$$\sum_{i=1}^{n} ciQi + (1-\alpha)ciQi \leq B \quad (2)$$

We employ a Lagrange's an multiplier λ to dualize the constraint and solve the problem as an unconstrained optimisation problem, obtaining the optimal order quantity $Q_i^*$ for each product. The prerequisites that must be met for optimality include

$$\frac{\partial Z}{\partial Qi} = 0$$

which imply that for each product i,

$$-\frac{DiSi}{(Qi)2} + \frac{hici}{2} + \frac{pi\gamma}{2} + (1-\alpha)ci(\delta - \gamma) + \lambda Ai = 0 \quad (3)$$

where $A_i = c_i + (1-\alpha_i)c_i$. Solving equation (3), we get

$$Q_i^* = \sqrt{\frac{2DiSi}{hici + pi\gamma + 2\lambda Ai + 2ci(1-\alpha)(\delta - \gamma)}} \quad (4)$$

Any iterative search method, such as interval bisection or optimisation software, can be used to find the ideal value of the Lagrange's an coefficient λ in equation (4).

## IV. Numerical example

Take three items, X, Y and Z whose yearly demands are, respectively, 20,000, 40,000 and 50,000 units. Products X, Y and Z have ordering costs of \$10, \$20 and \$40 respectively. For the three products, the proportion holding costs are 12%, 14% and 16% respectively, and the corresponding interest charged and generated are 12% and 14%. Product X has unit costs of \$6.25, Y has unit costs of \$7.80, while product Z has unit costs of \$9.40. Products X, Y and Z have selling prices of \$7.00, \$8.25, and \$9.75, respectively. For all three products, the lead times from order placement to payment and order reception are 40 and 30 days, respectively. Both the unpaid debt from the previous period and the total cost of the things ordered during this time frame should fall within the buyer's budget limit B of \$15 000. It should be noted that $\alpha$, which represents the portion of the entire order paid for in the current cycle, can have any value between 0 and 1. We attempt various values for $\alpha$ after beginning the example with $\alpha = 1$. Using Solver, we determined the ideal value of $\lambda$. The ideal order values for products X, Y and Z are 343.25, 608.24, and 862.81, respectively, based on equation (4) for $\alpha = 1$. The ideal cost of inventory

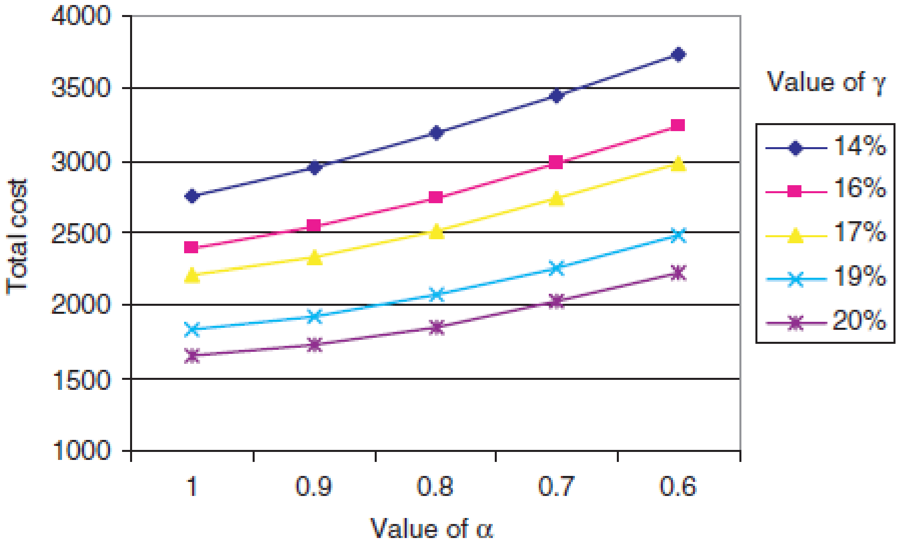


Figure 1 shows the total inventory costs for different alpha and gamma levels.

\$2,761.31 is the amount. Figure 1 shows the variations in the overall cost for different values of $\alpha$ and $\gamma$. Figure 1 makes it evident that, even in cases when the buyer has the option to pay after receiving the items, doing so will result in higher overall costs when only a portion of the purchase price is paid for and lower order numbers overall. It is also evident that when a partial payment is made, the overall cost rises, even in cases where the interest earned exceeds the interest charged. It is therefore advantageous to make the whole payment by the deadline.

## References

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