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Solution of the Fractional Order Airy's Type Partial Differential Equations by using Reduced Differential Transform Method

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Abstract: In this paper, we use Reduced Differential Transform Method (RDTM) to compute analytical and semi analytical approximate solutions of fractional order Airy's ordinary differential equations and fractional order Airy's and Airy's type partial differential equations subjected to certain initial boundary conditions. We compare the solutions obtain by RDTM with exact solutions . we find out convergent series solutions by taking different fractional values. RDTM gives reliability, effectiveness, efficiency, and strengthening of computed mathematical results in order to easily solve fractional order Airy's type differential equations.

Keywords: Reduced Differential Transform Method, Airy's fractional order Partial differential equations, Series solutions

Introduction:

Reduced Differential Transform Method is an alternative approach to overcome the demerit of complex calculation of Differential Transform Method, capable of reducing the size of the calculation. As a special advantage of Reduced Differential Transform Method rather than Differential Transform Method, the reduced differential transform recursive equations produce exactly all the Poisson series coefficients of solutions, whereas the differential transform recursive equations produce exactly all the Taylor series coefficients of solutions. We notice that the Reduced Differential Transform Method technique[11,12] is highly accurate, rapidly converge and is very easily implementable mathematical tool for the multidimensional physical problems emerging in various domains of engineering and allied sciences. Fractional order Airy's ordinary differential equations and time fractional order Airy's and Airy's type partial differential equations by applying the RDTM in the existing literature.[1,2,7,10]

Analysis of the method: Consider a function of two variables $f(s,t)$ and suppose that it can be represented as product of two single variables, i.e. $f(s,t) = f_1(t) f_2(t)$ on the basis of properties of differential transform the function $f(s,t)$ can be represented as

$$f(s,t) = \sum_{i=0}^{\infty} F_1(i)\, s^i \sum_{j=0}^{\infty} F_2(j) t^j = \sum_{k=0}^{\infty} U_k(x) t^k$$

The basic definitions of Reduced Differential Transform Method are introduced as follows.

Definition: If the function u(x, t) is analytic and differential continuously with respect to time t and space x in the domain of interest then let,

$$U_k(x) = \frac{1}{k!}\left[\frac{\partial^k u(x,t)}{\partial t^k}\right]_{t=0} \tag{1}$$

Where the t – dimensional spectrum function $U_k(x)$ is the transformed function. $U(x,t)$ represent transformed function.

Definition : The differential inverse transform of $U_k(x)$ is defined as ,

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)\, t^k \tag{2}$$

Then combining equations (1) and (2) we write

$$u(x,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k u(x,t)}{\partial t^k}\right]_{t=0} t^k \tag{3}$$

From the above definition, it can be found that the concept of Reduced Differential Transform Method is derived from the power series expansion. The fundamental operation performed by Reduced Differential Transform Method can be readily obtained and are listed in table1

Functional form of function	Transformed form of function
$u(x,t)$	$U_k(x)=\frac{1}{k!}\left[\frac{\partial^k u(x,t)}{\partial t^k}\right]_{t=0}$
$w(x,t)=u(x,t)\pm v(x,t)$	$W_k(x)=U_k(x)\pm V_k(x)$
$w(x,t)=\alpha\, u(x,t)$	$W_k(x)=\alpha\, U_k(x),\alpha$ is constant
$w(x,t)=x^m\, t^m$	$W_k(x)=x^m\delta(k-n),\delta(k)=1,k=0$ $=0,k\neq 0$
$w(x,t)=u(x,t)\, v(x,t)$	$W_k(x)=\sum_{r=0}^{k} U_r(x)V_{k-r}(x)$
$w(x,t)=\frac{\partial^r u(x,t)}{\partial t^r}$	$W_k(x)=(k+1)\ldots(k+r)\,U_{k+r}(x)$
$w(x,t)=\frac{\partial u(x,t)}{\partial x}$	$W_k(x)=\frac{\partial U_k(x)}{\partial x}$
$w(x,t)=\frac{\partial^2 u(x,t)}{\partial x^2}$	$W_k(x)=\frac{\partial^2 U_k(x)}{\partial x^2}$
$f(s,t)=\sin\,(a\,s+b\,t),$	$F_k(x)=\frac{a^k}{k!}\sin\left(\frac{k\,\pi}{2}+a\,s\right)$
$f(s,t)=\cos\,(a\,s+b\,t),$	$F_k(x)=\frac{a^n}{k!}\cos\left(\frac{k\,\pi}{2}+a\,s\right)$
$f(s,t)=e^{a\,x+b\,t}$	$F_k(x)=\frac{b^k}{k!}\,e^{a\,x}$

Table (1)

Time Fractional Airy's Equations: Consider one dimensional fractional Airy's partial differential equation in Caputo sense

$$\frac{\partial^{\propto}}{\partial t^{\propto}}u(x,t)=\beta\frac{\partial^3}{\partial x^3}u(x,t), x\in R, t>0,\ \ 0<\propto<1 \tag{4}$$

where $\beta=\pm 1$ subjected to the conditions $u(x,0)=\emptyset(x),\ x\epsilon R$ (5)

Applying RDTM on (4),(5) we get

$$R_D\left[\frac{\partial^{\propto}}{\partial t^{\propto}}u(x,t)\right]=R_D\left[\beta\frac{\partial^3}{\partial x^3}u(x,t)\right] \tag{6}$$

and

$$R_D[u(x,0)] = R_D[\emptyset(x)] \tag{7}$$

By applying table(1) we get,

$$U_{k+1}(x) = \beta \frac{\Gamma(k\alpha+1)}{\Gamma(k+1)(\alpha+1)}\left[\frac{\partial^3}{\partial x^3}U_k(x)\right], x \in R,\ k = 0,1,2... \tag{8}$$

$$U_0(x) = \phi(x), x \in R \tag{9}$$

Putting (9) in (8) we get

$$\text{For } k = 0, U_1(x) = \beta \frac{1}{\Gamma(\alpha+1)}\left[\frac{\partial^3}{\partial x^3}\phi(x)\right]$$

$$\text{For } k = 1, U_2(x) = \beta^2 \frac{1}{\Gamma(2\alpha+1)}\left[\frac{\partial^6}{\partial x^6}\phi(x)\right],\ldots$$

Putting these equations in (2), we get

$$U(x,t) = \sum_{k=0}^{\infty} U_k(x)t^{k\alpha}, t \succ 0$$

Example1:Consider one dimensional time fractional Airy partial differential equation for $\beta = 1$

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{\partial^3 u}{\partial x^3} x \in R, t \succ 0, 0 \le \alpha \le 1 \tag{10}$$

subjected to the condition

$$u(x,0) = \cos \pi x + e^{\pi x} \tag{11}$$

Applying RDTM on equations (10,11) we get

$$U_{k+1}(x) = \beta \frac{\Gamma(k\alpha+1)}{\Gamma(k\alpha+\alpha+1)}\left[\frac{\partial^3}{\partial x^3}U_k(x)\right], x \in R,\ k = 0,1,2... \tag{12}$$

Using RDTM on (11)

$$u(x,0) = u_0(x) = \cos \pi x + e^{\pi x}, x \in R \tag{13}$$

Using equations (12) and (13)we get $U_k(x)$

$$U_1(x) = \frac{\pi^3\left(\sin \pi x + e^{\pi x}\right)}{\Gamma(\alpha+1)}$$

$$U_2(x) = \frac{-\pi^6\left(\cos \pi x - e^{\pi x}\right)}{\Gamma(2\alpha+1)}$$

$$U_3(x) = \frac{-\pi^9\left(\sin \pi x - e^{\pi x}\right)}{\Gamma(3\alpha+1)}, \ldots$$

Thus the fractional differential inverse transform of $U_k(x)$ gives

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x) t^{k\alpha}$$

$$u(x,t) = \left(\cos \pi x + e^{\pi x}\right) + \frac{\pi^3\left(\sin \pi x + e^{\pi x}\right)}{1!} t^1 - \frac{\pi^6\left(\cos \pi x - e^{\pi x}\right)}{2!} t^2 - \frac{\pi^9\left(\sin \pi x + e^{\pi x}\right)}{3!} t^3 + \ldots$$

For $\alpha = \frac{1}{2}$

$$u(x,t) = \left(\cos \pi x + e^{\pi x}\right) + \frac{\pi^3\left(\sin \pi x + e^{\pi x}\right)}{1!} t^{\frac{1}{2}} - \frac{\pi^6\left(\cos \pi x - e^{\pi x}\right)}{2!} t^1 - \frac{\pi^9\left(\sin \pi x + e^{\pi x}\right)}{3!} t^{\frac{3}{2}} + \ldots$$

Example 2 Consider one dimensional time fractional Airy partial differential equation for $\beta = 1$

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{\partial^3 u}{\partial x^3} \quad x \in R, t \geq 0, 0 \leq \alpha \leq 1 \tag{14}$$

subjected to the condition $u(x,0) = \frac{1}{6} x^3$ (15)

Applying RDTM on equations (14,15) we get

$$U_{k+1}(x) = \beta \, \frac{\Gamma(k\alpha+1)}{\Gamma(k\alpha+\alpha+1)} \left[\frac{\partial^3}{\partial x^3} U_k(x) \right], x \in R, \; k = 0,1,2\ldots \tag{16}$$

$$u_0(x) = \frac{1}{6} x^3 \tag{17}$$

Using equations (16) and (17)we get $U_k(x)$

$$U_1(x) = \frac{1}{\Gamma(\alpha+1)}$$

For k = 1, $U_2(x) = \frac{\Gamma(\alpha+1)}{\Gamma(2\alpha+1)} \left[\frac{\partial^3}{\partial x^3} U_1(x) \right]$

For k = 2, $U_3(x) = 0, \ldots \quad , U_k(x) = 0$

Thus the fractional differential inverse transform of $U_k(x)$ gives

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x) t^{k\alpha} = \frac{1}{6} x^3 + \frac{1}{\Gamma(\alpha+1)} t^\alpha$$

For $\alpha = 1,\ u(x,t) = \frac{1}{6}x^3 + t$

Conclusion: In this study we have discussed the reduced differential transform method to find the solution of one dimensional time fractional Airy's and Airy's type partial differential equation based on the basic Caputo's definition of fractional derivatives. The results show that the RDTM technique gives the approximate series solutions. The accuracy of the solutions can be improve by increasing number of terms in series solutions. The techniques used in this work can also be applied to solve linear and non-linear time fractional partial differential equation and multi-dimensional physical problems emerging in various fields of engineering and applied sciences.

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