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SOME RESULTS ON APPROXIMATE FIXED-POINT THEOREM

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Abstract: In this paper, Some Results on approximate Fixed-points are proved results are believed to be new.

Index Terms- Fixed-point, Approximate Fixed Point, b-metric Space

1. Introduction: -

Fixed-point theory is very useful for solving various problems in pure and applied mathematics.
"A fixed-point is a point which remains invariant under the transformation T.
i.e. $Tx = x, \forall x \in X$ if for any $z, d(Tz, z) \leq \epsilon, then\ z$ is called approximate fixed point of T.

First the Cromme and Diener have found Approximate fixed-points by generalizing Brouwer's fixed point theorem to a continuous map and finally $Tijs$,Torre and Branzi have discussed approximate fixed-point theorems for contractive and non-expansive type maps.

So, in this paper I have proved "Some Results for existence of ϵ-fixed-point for Kannan operator and Chatterjee operator.

2. Preliminaries

Definition 2.1: - Let $s \geq 1$ be a given real number. A function $d: X \times Y \to \mathbb{R}_+$ (Set of non-negative real numbers) is said to be a b-metric $\Leftrightarrow \forall x, y, z \in X$ the following conditions are satisfied. Let X be a non-empty set

(i) $d(x, y) = 0 \Leftrightarrow x = y$
(ii) $d(x, y) = d(y, x)$
(iii) $d(x, z) \leq s[d(x, y) + d(y, z)]$

A pair (X, d) is called a b-metric space.

Definition 2.2: - Let $T: X \to X, \epsilon > 0$ and $x_0 \in X$.Then the element $x_0 \in X$ is an approximate

fixed-point of T if $d(Tx_0, x_0) < \epsilon$.
T is said to satisfy approximate fixed-point property (AFPP) if for every $\epsilon > 0$.

$$\mathrm{Fix}_\epsilon(\mathrm{T}) \neq \phi$$

$$\mathrm{F}_\epsilon(\mathrm{T}) = \{ \mathrm{x} \in \mathrm{X} : \mathrm{x} \text{ is an } \epsilon - \text{Fixed point of T}\}$$

Definition 2.3: - A mapping $T: X \to X$ is a contraction if $\exists\, \alpha \in\,]0,1[$such that $d(Tx, Ty) \leq \alpha d(x, y);\ \forall x, y \in X$.

Definition 2.4: - A map $T: X \to X$ is said to be asymptotically regular if for any- $x \in X,\ \lim_{n\to\infty} d(T^n x, T^{n+1} x) = 0\ as\ n \to \infty, \forall x \in X,$
Then T has approximate fixed-point property.

Kannan Operator
A mapping $T: X \to X$ where (X, d) is a metric space is said to be a Kannan Type Contraction if
$d(Tx, Ty) \leq c\big(d(x, Tx) + d(y, Ty)\big) \quad \forall x, y \in X$ ……………….............(2.5)
Where $c \in (0, \frac{1}{2})$

Chatterjee Operator
A mapping $T: X \to X$ where (X, d) is a metric space is said to be a Chatterjee Type Contraction if ,
$d(Tx, Ty) \leq c\big(d(x, Ty) + d(y, Tx)\big) \quad \forall x, y \in X$ ………………... ……...(2.6)
Where $c \in (0,1)$

3. Main Result

Theorem 3.1: - Let (X, d) be a b-metric space and $T: X \to X$ satisfies (2.5) then T has approximate fixed point property.

Proof: - Let $\epsilon > 0$ and $x \in X$ then ,

$$d(T^n x, T^{n+1} x) = d(T(\,T^{n-1}\, x), T(\,T^n x))$$

$$\leq c[d(T^{n-1}x, T(T^{n-1}x)) + d\big(T^n x, T(T^n x)\big)] \quad \text{[by definition 2.5]}$$

$$\leq c\, d(T^{n-1}x, T^n x) + c\, d(T^n x, T^{n+1} x)$$

$$d(T^n x, T^{n+1} x) - c\, d(T^n x, T^{n+1} x) \leq c\, d(T^{n-1}x, T^n)$$

$$d(T^n x, T^{n+1} x)(1 - c) \leq c\, d(T^{n-1}x, T^n)$$

$$d(T^n x, T^{n+1} x) \leq \frac{c}{(1-c)} d(T^{n-1}x, T^n)$$

$$d(T^n x, T^{n+1} x) \leq \left(\frac{c}{1-c}\right)^n d(x, Tx)$$

$$\therefore \quad d(T^n x, T^{n+1} x) \to 0, \quad as\ n \to \infty\ \forall\ x \in X$$

$$\therefore \quad Fix_\epsilon(T) \neq \phi$$

Theorem 3.2: - Let (X, d) be a b-metric space and $T: X \to X$ a Kannan Operator. Then for each $\epsilon > 0$, the diameter of $Fix_\epsilon(T)$ is not larger then

$$s\epsilon[1 + s + 2sc]$$

Proof: - We know that T has the approximate fixed-point property. So, we can take x and y any two ϵ-fixed-point of T. Then

$$d(x, y) \leq s[\ d(x, Tx) + d(Tx, y)]$$

$$\leq s\ \epsilon + s\ d(Tx, y) \qquad \text{[by definition 2.2]}$$

$$\leq s\ \epsilon + s[s\{\ d(Tx, Ty) + d(Ty, y)\}] \quad \text{[by definition 2.1 (iii)]}$$

$$\leq s\ \epsilon + s^2\ d(Tx, Ty) + s^2 d(Ty, y)$$

$$\leq s\ \epsilon\ + s^2\epsilon + s^2\ d(Tx, Ty)$$

$$\leq s\ \epsilon\ + s^2\epsilon + s^2[c\ \{d(x, Tx) + d(y, Ty)\}] \ \text{[by definition 2.5]}$$

$$\leq s\ \epsilon\ + s^2\epsilon + s^2\epsilon c + s^2\epsilon c \qquad \text{[by definition 2.2]}$$

$$\leq s\ \epsilon\ + s^2\epsilon + 2s^2\epsilon c$$

$$\leq s\ \epsilon[1 + s + 2sc]$$

$$d(x, y) \leq s\ \epsilon[1 + s + 2sc]$$

This completes the proof.

Theorem 3.3: - Let (X, d) be a b-metric space and $T: X \to X$ satisfies (2.6) then T has approximate fixed point property.

Proof: - Let $\epsilon > 0$ and $x \in X$ then ,

$$d(T^n x, T^{n+1} x) = d(T(\ T^{n-1}\ x), T(\ T^n x))$$

$$\leq c[d(T^{n-1}x, T(T^n x)) + d\big(T^n x, T(T^{n-1}x)\big)] \qquad \text{[by definition 2.6]}$$

$$\leq c\ d(T^{n-1}x, T^{n+1}x) + c\ d(T^n x, T^n x)$$

$$\leq c\ d(T^{n-1}x, T^{n+1}x) + 0$$

$$d(T^n x, T^{n+1} x)\ \leq c\ d(T^{n-1}x, T^n x) + c\ d(T^n x, T^{n+1} x) \qquad \text{[by definition 2.1(iii)]}$$

$$d(T^n x, T^{n+1} x) - c\ d(T^n x, T^{n+1} x) \leq c\ d(T^{n-1}x, T^n)$$

$$d(T^n x, T^{n+1}x)(1-c) \leq c\, d(T^{n-1}x, T^n)$$

$$d(T^n x, T^{n+1}x) \leq \frac{c}{(1-c)} d(T^{n-1}x, T^n)$$

$$d(T^n x, T^{n+1}x) \leq \left(\frac{c}{1-c} \right)^n d(x, Tx)$$

$$\therefore \quad d(T^n x, T^{n+1}x) \to 0, \quad as\, n \to \infty \;\; \forall\, x \in X$$

$$\therefore \quad Fix_\epsilon(T) \neq \phi$$

Theorem 3.4: - Let (X, d) be a b-metric space and $T: X \to X$ a Chatterjee Operator. Then for each $\epsilon > 0$, the diameter of $Fix_\epsilon(T)$ is not larger then

$$\frac{s\epsilon[1 + s + 2s^2c]}{(1 - 2s^3c)}$$

Proof: - We know that T has the approximate fixed-point property. So, we can take x and y any two ϵ-fixed-point of T. Then

$$d(x,y) \leq s[\, d(x,Tx) + d(Tx,y)]$$

$$\leq s\, d(x,Tx) + s\, d(Tx,y)$$

$$\leq s\,\epsilon + s\, d(Tx,y) \qquad \text{[by definition 2.2]}$$

$$\leq s\epsilon + s[s\{\, d(Tx,Ty) + d(Ty,y)\}] \qquad \text{[by definition 2.1(iii)]}$$

$$\leq s\epsilon + s^2 d(Tx,Ty) + s^2 d(Ty,y)$$

$$\leq s\epsilon + s^2\epsilon + s^2 d(Tx,Ty) \qquad \text{[by definition 2.2]}$$

$$\leq s\epsilon + s^2\epsilon + s^2[c\{d(x,Ty) + d(y,Tx)\}] \qquad \text{[by definition 2.6]}$$

$$\leq s\epsilon + s^2\epsilon + s^2c[s\{d(x,y) + d(y,Ty)\} + s\{d(y,x) + d(x,Tx\}]$$

[by definition 2.1(iii)]

$$\leq s\epsilon + s^2\epsilon + s^3c\, d(x,y) + s^3c\epsilon + s^3c\, d(x,y) + s^3c\epsilon \qquad \text{[by definition 2.2]}$$

$$\leq s\epsilon + s^2\epsilon + 2\, s^3c\, d(x,y) + 2\, s^3c\epsilon$$

$$d(x,y)(1 - 2\, s^3c) \leq s\epsilon + s^2\epsilon + 2\, s^3c\epsilon$$

$$d(x, y) \leq \frac{s\epsilon[1 + s + 2s^2c]}{(1 - 2s^3c)}$$

This completes the proof.

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