



## Application of Calculus in Life Science

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### Abstract

In this paper we are going to familiar with the calculation and derive the general Euler Lagrange's equation for functional that depend on function of one variable. We have the calculation of variation has traditionally applied in solving the problem of Mechanics we apply in biology by means of minime surface we familiar with the idea of using space curves to model protein structure and at the last we conclude the free energy associated with these structures by deriving two Euler- Lagrange equation depend on curvature .

### Introduction

#### Calculus of variation

Calculus of variation help to formulate Geodesic problems on a plane sphere. There are many laws of physics which are written as variational principles. The principle of least action is equivalent to Newton Second law of motion in mechanical system It leads naturally formulation of mechanics. It is a highly important topic on the Life science.

$$E [t] = \int_a^b f (t, y(t), y'(t))dt$$

Where the integrand  $F[t, y(t), y' (t)]$  is a function of the independent variable  $t$  a function  $y(t)$  and the first derivative  $y'(t)$  with prime notation denoting the derivative with respect to  $t$ . The function  $y(t)$  is on  $D$ , the space of all  $C^1$  function defined on the  $[a,b]$  with  $y(a) = A$  and  $y(b) = B$  for any  $y(t) \in D$

#### Euler - Lagrange equation

Let  $E_p$  be the space of  $C^2$  curves  $x: [0,1] \rightarrow \mathbb{R}^n$  with  $x(0)=f$  and  $x(1)= Q$ . Let  $L: \mathbb{R}^{2n+1} \rightarrow \mathbb{R}$  be a sufficiently differentiable function and let us consider the functional  $S: E_p \rightarrow \mathbb{R}$

defined by

$$S [x] = \int_0^1 L ( x(t), x'(t))dt$$

The function  $L$  is called the Lagrangian and the functional  $S$  is called the action Extremizing  $S$  all yield a differential equation for  $x$ . Recall that a path  $x$  is a Critical point for the action if for all endpoint - fixed Variations  $E$ , we have

$$\frac{d}{ds} S [x+se] = 0$$

$S=0$

Differentially under the integral

Sign, we find

$$\begin{aligned} 0 &= \int_0^1 \frac{dl}{dx} (x + s \in, t) dt \quad s=0 \\ &= \int_0^1 \left( \sum_{i=1}^n \frac{dl}{dxi} \frac{dlei}{dxi} \right) dt \\ &= \int_0^1 \left( \sum_{i=1}^n \frac{dl}{dxi} - \frac{d}{dt} \frac{dl}{dxi} \right) \in i dt \\ &= \int_0^1 \cdot \sum_{i=1}^n \left( \frac{dl}{dxi} - \frac{d}{dt} \frac{dl}{dxi} \right) \in i dt \end{aligned}$$

Where we have integrated by parts and used that  $\in (0) = \in (1)$

Using the fundamental lemma

This is equivalent to  $\frac{d}{dx^2} - \frac{d}{dt} \frac{dl}{dx^2}$

For all  $l = 1, 2, \dots, n$

This is the Euler-Lagrangs equation

\* Minimal surface

Minimal Surfaces are defined as Surfaces with zero mean Curvature. Minimal Surfaces may also be characterized as surfaces of minimal surface area for given boundary Condition.

A plane is a trivial minimal Surface and the first non-trivial examples (the Catenoid and helicoid) were founded by me usnier in 1776-

A Surface can be parameterized using an isothermal parameterization. The term minimal Surface is used because these surface Originally areas as Surfaces that minimized total surface area subject to some Constraints.



Figure 3.4: A soap film of a helicoid. Image courtesy of <http://www.math.cornell.edu/~mec/Summer2009/Fok/index.html>

## quaternary

\* Regular Secondary Structure of protein.

Protein Secondary Structure is the local Spatial Conformation of the polypeptide back bone excluding the Side Chains. The two most common Secondary Structure elements are alpha helices and beta sheet. Though beta turns and omega loop occur as well. Secondary structure elements typically spontaneously form an intricate before the protein folds into its three dimensional tertiary Structure.

Secondary Structure is formally determined by the pattern of hydrogen bonds between the amine hydrogen and Carboxyl oxygen. Atoms in the peptide backbone. Secondary structure may alternately be defined based on the regular pattern of backbone dihedral angles in a particular region of the Ramchandran plot regardless of whether it has the correct hydrogen bonds. The concept of secondary structure was first introduced by

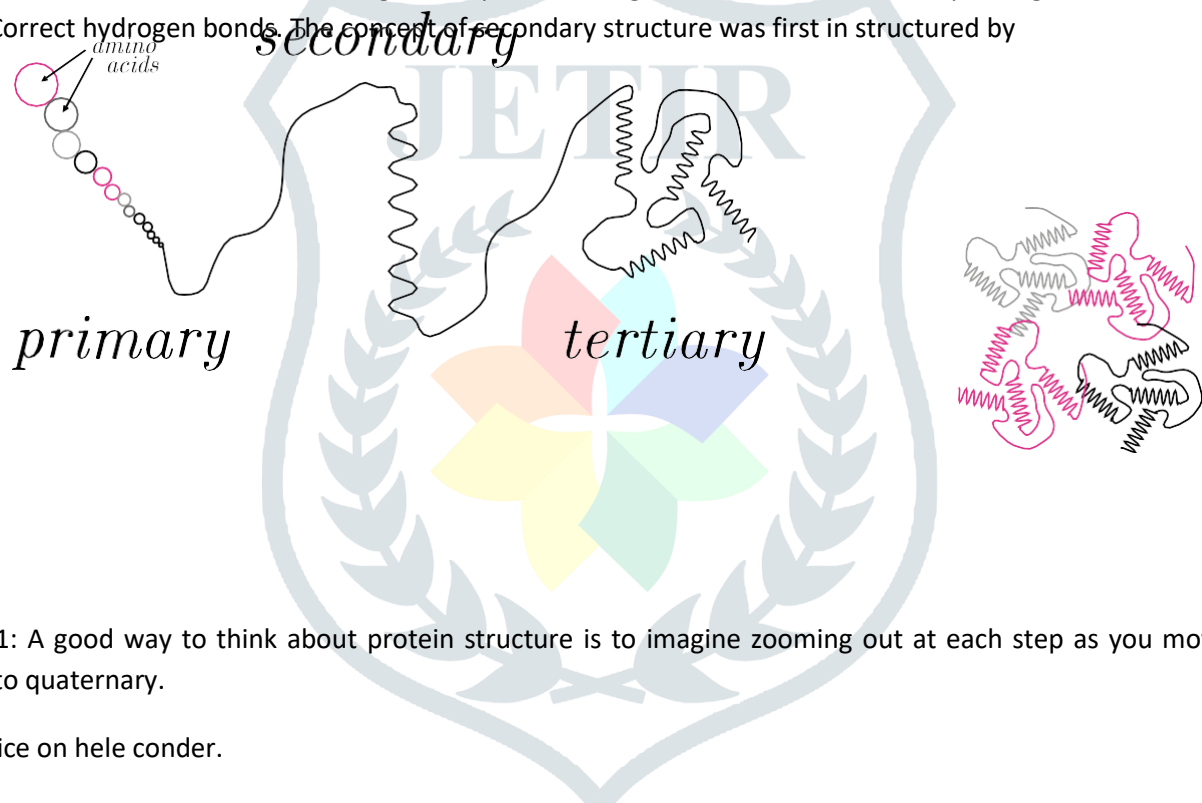


figure 3.1: A good way to think about protein structure is to imagine zooming out at each step as you move from primary to quaternary.

Geodesic on helicoid.

A geodesic on a Helicoid is a curve that follows the shortest path between two points on the surface of the Helicoid. It can also be described as a curve that is locally straight and geodesic curvilinear

Minimal surface

We have

$$S[J] = \iint F(x, y, z, g_x, g_y) dx dy$$

$$\iint \sqrt{1 + g_x^2 + g_y^2} dx dy$$

Among admissible surface

$$Z = g(x, y, a)$$

The associated Euler-Lagrange equation is

$$\frac{\partial f}{\partial g} - \frac{d}{dx} \frac{\partial f}{\partial g_x} - \frac{d}{dy} \frac{\partial f}{\partial g_y} = 0$$

We have F does not depend explicitly on g , so the abonesimpli fier to

$$\frac{d}{dx} \frac{\partial f}{\partial g_x} + \frac{d}{dy} \frac{\partial f}{\partial g_y} = 0$$

Company the appropriated partial derivatives , plugging them into gn

$$\begin{aligned} \frac{d}{dx} \left[ \frac{g_x}{\sqrt{1+g_x^2+g_y^2}} \right] + \frac{d}{dy} \left[ \frac{g_y}{\sqrt{1+g_x^2+g_y^2}} \right] &= 0 \\ \frac{g_{xx}\sqrt{1+g_x^2+g_y^2} - g_x \left[ \frac{g_x g_{xx} + g_y g_{xy}}{\sqrt{1+g_x^2+g_y^2}} \right] + g_{yy}\sqrt{1+g_x^2+g_y^2} - g_y \left[ \frac{g_x g_{xy} + g_y g_{yy}}{\sqrt{1+g_x^2+g_y^2}} \right]}{1+g_x^2+g_y^2} &= 0 \\ \frac{g_{xx}(1+g_x^2+g_y^2) - g_x^2 g_{xx} - g_x g_y g_{xy} + g_{yy}(1+g_x^2+g_y^2) - g_x g_y g_{xy} - g_y^2 g_{yy}}{\sqrt{1+g_x^2+g_y^2}} &= 0 \\ (1+g_y^2)g_{xx} - 2g_x g_y g_{xy} + (1+g_x^2)g_{yy} &= 0, \end{aligned}$$

which is the Minimal Surface Equation for the graph g.

The helicoids

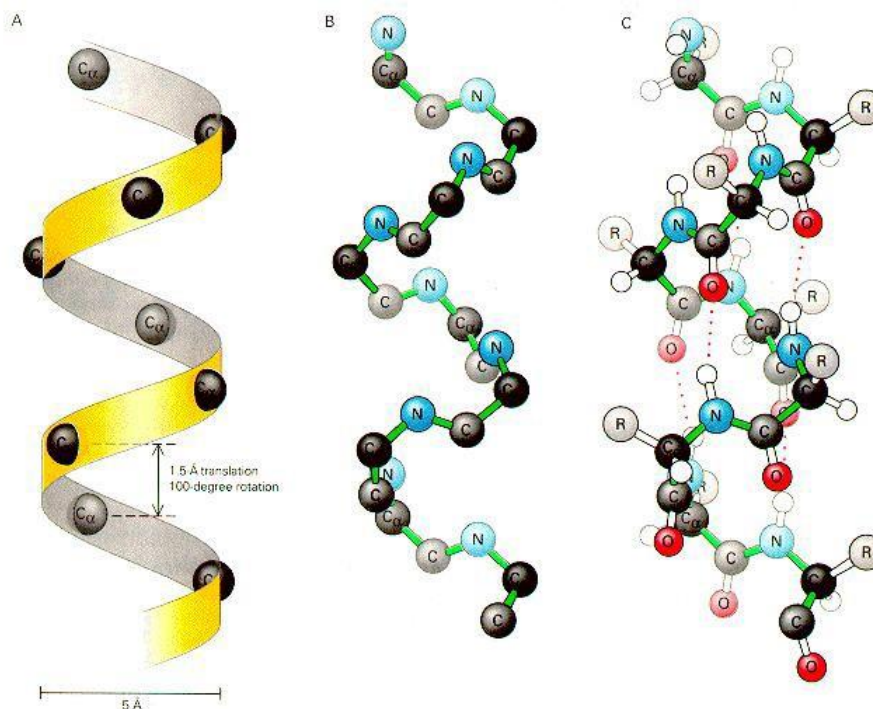
Let as consrdiv one parameterization

$$\bar{x}(u,v) = (-b \sin hv \sin u, b \sin hv \sin v, bu)$$

Where b ∈ R is an arbitrary constant the lengent vectors associated with this parameterization of the heli coud are

$$\bar{x}_u = (-b \sin hv \cos u - b \sin hv \sin v)$$

$$\bar{x}_v = (-b \cos hv \sin u, b \cos hv \cos u, 0)$$



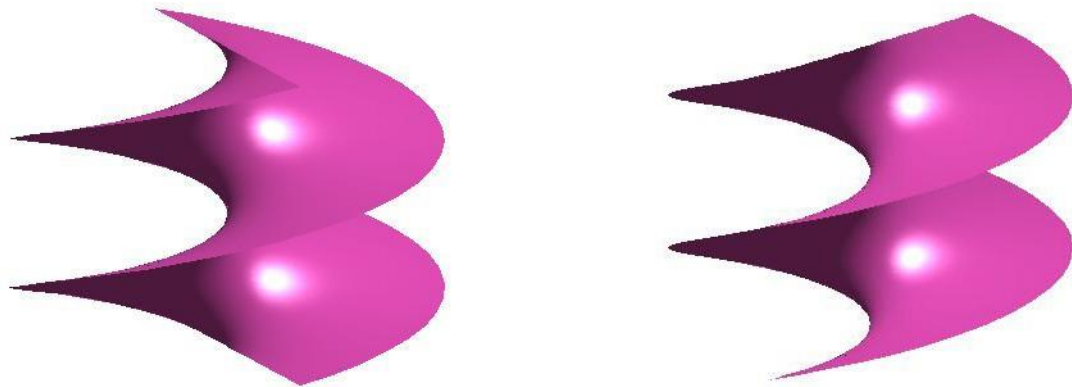


Figure 3.2: The  $\alpha$ -helix. Image courtesy of cmgm.stanford.edu

Figure 3.3: The helicoid

### Conclusion

This Clear from basic results of Calculus of variation namely the Simplest Euler-Lagrange equation and have examined the Connection to minimal surface In Considering minimal Surface, we said that the lone between the helicoid and elocs. One of the most Common repeating units of protein Structure and then extended this Connection to derive two Euler-Lagrange. Equations which are related to potential free energy functional

$$E(x) = \int F(K) dL \text{ of Protein Structure}$$

We can use these two Euler-Lagrange equation to derive different discussion of Some possible solution. The article Me Coy [Me co<sub>8</sub>] Natunchy we could to want restrict our self F(k) that admit helics as minimizing solution to the energy functional E(x) there solution Could shed Some light on protein Structure different angle, which are current a Ve nues of research

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### REFERENCES

- [AB] M. Ashbaugh and R. Benguri, *The problem of queen dido: Overview of the subject of isoperimetry.*
- [BF09] M. Barros and A. Ferrández, *A conformal variational approach for helices in nature*, Journal of Mathematical Physics 50 (2009).
- [FNS05] A. Feoli, V. V. Nesterenko, and G. Scarpetta, *Functionals linear in curvature and statistics of helical proteins*, Nuclear Physics B 705 (2005), 577–592.
- [GH96] M. Giaquinta and S. Hildebrandt, *Calculus of variations*, Springer, 1996.
- [HMT08] J. Hill, J. McCoy, and N. Thamwattana, *Energy density functions for protein structures*, Quarterly Journal of Mechanics and Applied Mathematics 61 (2008).

- [LS82] A.H. Louie and R.L. Somorjai, *Differential geometry of proteins: a structural and dynamical representation of patterns*, Journal of Theoretical Biology (1982).
- [McC08] J. McCoy, *Helices for mathematical modelling of proteins, nucleic acids and polymers*, Journal of Mathematical Analysis and Applications 347 (2008), 255–265.

