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Algorithmic Study of the Grid Methods for Numerical Solution

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Abstract: **Solution is obtained with the help of some approximate numerical grid methods using a computer in computational method. Generally, grid methods are used to obtain solution in computational method. Computational grid method of analysis are applicable to a much wider class of mathematical formulations and hence most problems can be solved. With the advent of fast computers, algorithmic models have become very widely used valuable tool for solving engineering problems.**

Index Terms **- Approximate; Grid; Numerical; Mathematical model; Algorithmic.**

1. INTRODUCTION

Many practical problems are complex in nature. A model is often needed to solve a system consisting of any domain of interest and various phenomena that take place in it. Modelling can be done through physical means with laboratory and field experiments or through mathematical means. Physical modelling in laboratory or field can be expressive as well as time consuming. Due to these difficulties in physical modelling in laboratories or in the real world, use of mathematics in solving the problems has becomes widespread in the recent times. Depending on the mathematical modelling approach, the development of numerous numerical methodologies have made the mathematical modelling invertible for all branches of engineering.

There are variety of mathematical models are available which are based on the numerical methods such as Method of Characteristics, Finite Difference Method (FDM), Finite Volume Method (FVM), Finite Element Method (FEM), Boundary Element Method (BEM) and other have been developed by mathematician and scientists for solution of engineering problems. Out of many available numerical methods, FDM, FVM, FEM and BEM are most popular numerical grid techniques. When numerical grid methods are used in mathematical modelling, the ensuring computer model generally consists of pre-processing, processing and post-processing. In the pre-processing part of the model, input data required to solve the problem include domain geometry, initial and boundary conditions, coefficients and constants for the particular problem, value of universal constants, the grid information such as triangular grid, rectangular grid and so on and various options for the concerned problem like 1D, 2D and 3D steady or unsteady state analysis. Grid generation is an important task of pre-processing. The grid generation includes discretization of the domain into elements and determining grid points and it depends on the type of numerical method used and dimension of the model. Grid can be generated manually or computer programming can be written

or separate grid generating packages can be used. In the processing part, real simulation of the problem with the concerned numerical model is done which includes generation of element matrices, assembly of element equations, and imposition of boundary conditions and solution of system of equation. In the post-processing part, results obtained are processed in terms of tables, charts, graphs and so on. Computer program can be written such that results are displayed graphically or separate post-processing packages can be used.

The remainder of the review article is organized as follows. In section 2, what kind of the research work have been done by researchers are given. The section 3 gives the idea about the problem formulation. In section 4, the numerical grid methods and the essential basic steps have discussed. In section 5, we have mentioned what kind of computer resources, programs and separate packages are going to use. We have discussed the applications of numerical grid methods in section 6. Finally, section 7 concludes the review article.

2. LITERATURE SURVEY

Many Approximate numerical methods have evolved in the last six decades for solution of complex engineering problems due to advent of high speed computers. Out of various available numerical grid techniques FDM, FVM, FEM, BEM and mesh less have become popular among scientists and engineers. Out of these numerical grid methods, one of the most important advance in the field of numerical methods was the development of FEM in the 1940s.

Hrenikoff (1941) proposed the frame work method for the solution of elasticity problems. On the other side, Courant (1943) presented an assemblage of piecewise polynomial interpolation over triangular elements and the principal of minimum potential energy to solve torsion problems. Some mathematical aspects related to eigen values were developed for boundary value problems by Poyla (1954), Hersch (1955) and Weinberger (1958). Foundation of finite element was laid by Argyris in (1955) through his book on Energy Theorem and Structural Analysis. Turner and team (1956) developed the stiffness matrices for truss, beam and other elements for engineering analysis of structures.

For the first time in 1960, the terminology Finite Element Method was used by Clough (1960) in his paper on the plane elasticity. In 1960s, a large number of papers appeared related to the applications and developments of the finite element method. Curve edged elements of quadrilateral type were first introduced in FEM by Taig (1961). A number of international conference related to FEM were organized and the method got established. The first book on FEM was published by Zienkiewicz and Cheung in 1967. The procedure of deriving interpolation functions and its properties is very similar to that derived for quadratic type interpolation functions earlier. The scheme would be initially demonstrated for linear, quadratic and cubic type line elements before explaining further to higher order elements (Heubner, 1975). Pinder and Gray (1977), Wang and Anderson (1982), Huyakon and Pinder (1983), Segerlind (1984) and Istok (1989) have given more details on application of FEM to groundwater flow and transport problems.

For structural mechanics problems, vibrational functional or some kind of relationship useful in FEM analysis can be derived using the principal of virtual work, principle of minimum potential energy method is most commonly used (Shames and Dym, 1991 and Zienkiewicz) and Castigliano theorem (1991). It was also derived by employing direct equilibrium approach (Lohan, 1993). Finite element formulation for the parallel loading has already been presented in the book on plates by Reddy (1999) for detailed studies. The detailed discussion of non-linear analysis can be found in advanced finite element textbook such as Zienkiewicz and Taylor, Bathe (2001). Chapra and Canale (2002) presented the finite element formulations of some important steady-state partial differential equations governing the field problems in two dimensional Laplace equation is an elliptic equation. Based on the Galerkin FEM formulation, a computer program has been developed using the FORTRAN for the Laplace equation. Arpita M. (2008) have given more details on application on FEM-GA for groundwater remediation using optimization model. A Srirekha and Kusum Bashetty (2010) presented an overview on FEA and which is one of the application of FEM in health science. E. Nadal and team (2013) were proposed efficient finite element methodology based on Cartesian grids. Neela Nataraj and A. S. Vasudeva (2019) Murthy presented a summary of major contributions of Indian mathematicians to the mathematical aspects of the finite element method in the last one decade and that review is accessible to anybody with a background in partial differential equations and numerical techniques for solving it. By now, a large number of research papers, proceedings of international conferences and short-term courses and books have been published on the subject of FEM. Many software packages are also available to deal with various types of problems.

3. PROBLEM FORMULATION

Generally, the governing equations can be classified into algebraic equations, ordinary differential equations (ODE), partial differential equations (PDE) and integral equations. Differential equations are classified according to their order. An equation is said to be of order n if the highest derivative of the dependent variable is of order n.

As many physical laws are expressed in terms of the rate of change of a quantity rather than the magnitude itself, the ODE are of most importance in engineering practice. Let us consider, the deflection y of a mass spring system with damping can be expressed with a second order differential equation as

$$m\frac{d^2y}{dt^2} + c\frac{dy}{dt} + ky = 0 \qquad (1)$$

Where m is the mass, c is the damping coefficient, k is the spring coefficient and t is the time.

Dependent physical quantities vary with two or more independent variables and hence PDE formulations are used in many practical situations. The order of a PDE is that of the highest order partial derivative that appears in the equation.

Let us consider, a second order PDE with two independent variables as

$$a\frac{\partial^2 p}{\partial x^2} + b\frac{\partial^2 p}{\partial x \partial y} + c\frac{\partial^2 p}{\partial y^2} + \alpha = 0 \qquad (2)$$

Where a, b and c are functions of x and y and α is a function of x, y, p, $\frac{\partial p}{\partial x}$ and $\frac{\partial p}{\partial y}$.

Sometimes, physical laws are expressed in terms of the integral of one or more variable over the entire domain, and hence integral equations are of great importance in engineering practice. The integral equations can be expressed as

$$H(x) = \int g(x,t)f(t)dt + f(x)$$

$$F(x) = \int_a^b g(x,u)du + h(x)$$

3.1. INITIAL AND BOUNDARY CONDITIONS

Solution of ODE, PDE or integral equations are attempted in conjunction with initial and boundary conditions.

Consider the PDE which represented by equation (2) in a two dimensional domain and satisfy the homogeneous conditions on the boundary.

Essential or Dirichlet boundary condition

$$p = f_1$$

Natural or Neumann boundary condition

$$\frac{\partial p}{\partial n} = f_2$$

Mixed boundary condition

$$\frac{\partial p}{\partial n} + k(s)p = f_3$$

Where f_1, f_2 and f_3 are known values on the boundary, s is the arc length along boundary, measured from some fixed point on boundary and $\frac{\partial p}{\partial n}$ represents the differentiation along the outward normal n to boundary.

4. SOLUTION METHODOLOGIES

Analytical solution can be derived only for a small class of formulation involving ODE, PDE or integral equations. Generally, approximate methods such as numerical grid methods are used to obtain solution.

4.1. FINITE DIFFERENCE METHOD

Continuous variation of the function concerned is represented by a set of values at points on a grid of intersecting lines in the FDM. The problems to which the method applied are specified by a PDF, a solution region and boundary conditions. The FDM involves four basic steps.

- The solution region is divided into a grid of nodes.
- Grids are arranged in a rectangular mesh.
- Approximate the PDE and boundary conditions by a system of linear algebraic equations on grids.
- Solve the system of linear algebraic equations using Matrix algorithm.

4.2. FINITE VOLUME METHOD

The finite volume method (FVM) was originally developed as a special finite difference formulation and it is a discretization technique for PDE, especially those that arise from physical conservation laws. FVM uses a volume integral formulation of the problem with a finite partitioning set of volumes to discretize the equations. FVM is in common use for discretizing computational fluid dynamics equations. The FVM involves four basic steps.

- The solution region is divided into discrete control volumes.
- The integration of the governing equation over a control volume to yield a discretized equation at its nodal point.
- Discretized equations must be set up at each of the nodal points in order to solve a problem.
- Solve the resulting system of linear algebraic equations using Matrix algorithm.

4.3. FINITE ELEMENT METHOD

The finite element method (FEM) is a numerical grid technique for solving PDE. FEM is used widely for solving engineering problems in solid mechanics, heat transfer, structural mechanics, aerospace, automobiles, biomechanics, fluid mechanics and so on. FEM is one of the most flexible and versatile grid method for solving engineering problems. The FEM involves four basic steps.

- The solution region is divided into a finite number of elements such as triangular, quadrilateral shape.
- Derive governing equations for a particular element and determine the element coefficient matrix.
- Assemble all elements in the solution region to obtain the global coefficient matrix.
- Solve the resulting system of linear algebraic equations using Matrix algorithm.

4.4. BOUNDARY ELEMENT METHOD

PDE describing the solution region is transformed into an integral equation relating only to boundary values in the BEM. The BEM involves some important features as compared to other numerical grid method as

- It is based on Green's Integral Theorem.
- The boundary is discretized instead of the solution region.
- The computational dimension of the problem is reduced by one.
- It is ideally suited for solution of many 2D and 3D problems.

4.5. OTHER NUMERICAL METHODS

- **Gradient Discretization Method:** The gradient discretization method (GDM) is a numerical technique that encompasses a few standard or recent methods.
- **Spectral Method:** The idea is to write the solution of the differential equation as a sum of certain basis functions and then to choose the coefficients in the sum that best satisfy the differential equation.
- **Multi grid Method**: The idea is to accelerate the convergence of a basic iterative method by a global correction of the fine grid solution approximation from time to time, accomplished by solving a coarse problem.
- **Mesh free Method:** This method does not require a mesh or grid connecting the data points of the simulation domain. It is enable the simulation of some otherwise difficult types of problems, at the cost of extra computing time and programming effort.
- **Domain Decomposition Method:** It is used to solve a boundary value problem by splitting it into smaller boundary value problems on subdomains and iterating to coordinate the solution between adjacent subdomains.

5. SOFTWARE AND WEB-RESOURCES

Numerical analysis is a branch of mathematics that solves continuous problems using numeric approximation. It involves designing methods that give approximate but accurate numeric solutions, which is useful in cases where the exact solution is impossible or prohibitively expensive to calculate. Numerical analysis also involves characterizing the convergence, accuracy, stability, and computational complexity of these methods.

- **Python:** Program the numerical methods to create simple and efficient Python codes that output the numerical solutions at the required degree of accuracy. Create and manipulate arrays (vectors and matrices) by using NumPy. Use the plotting functions of matplotlib to present your results graphically.
- **R Programming:** It is used for numerical analysis and provides a solid introduction to the most useful numerical methods for scientific and engineering data analysis.
- **MATLAB**: It is widely used for applied numerical analysis in engineering, computational finance, and computational biology. It provides a range of numerical methods for interpolation, extrapolation, regression, differentiation and integration, linear systems of equations, eigenvalues and singular values, ODEs and PDEs.
- **C and C++ Programming:** It is well-organized and comprehensive programming language and its aims at enhancing and strengthening numerical methods concepts and a fast emerging preferred programming language among software developers.
- **Separate Software Packages**: In this section, some of the commonly used software packages in engineering have been discussed.
    - **STAAD** family of FEM analysis and design packages can be used for structure such as multi-stored building, towers, culvert, plants, bridges, stadiums and so on.
    - **PLAXIS** is a range of FEA packages for 2D and 3D analysis of foundation, deformation, stability, groundwater flow and so on in geotechnical engineering.
    - **ANSYS CFX** is based on CFD technology for simulations and solutions of steady state or transient analysis, laminar or turbulent flow analysis for a variety of problem.
    - **COMSOL Multi-Physics** formerly known as FEMLAB is a FEA and solver software package for various physics and engineering applications.
    - **FLEXPDE6** is multi-physics finite element solution environment for PDE and used for 1D, 2D and 3D PDE problems of heat flow, stress analysis, fluid mechanics, chemical reactions, electromagnetics, diffusion and so on. It builds a coupling matrix and solver it and plots the results.
    - **SFEAP** Software will provided sufficient background of learning and in reinforcing the understanding of FEM.

6. APPLICATIONS

Majority of applications of grid methods are in solid mechanics, fluid mechanics, electrical and electromagnetic problems as well as in bioengineering problems and so on. A brief description of the applications of grid methods in various engineering filed as

- Grid method applications in equilibrium conditions includes analysis of beams, plates, shell structure, stress and torsion of various structures.
- Grid method applications include slope stability analysis, soil structure interactions, seepage of fluids in soils and rocks, analysis of dams, tunnels, bore holes.
- Grid method applications include steady and transient seepage in aquifers and porous media, movement of fluids in containers, external and internal flow analysis, ocean and harbors, salinity and pollution studies in surface and sub-surface water problems.
- Grid method applications include steady and transient thermal analysis is solids and fluids, stress analysis in solids, automotive design and analysis and manufacturing process simulation.
- It is used in electrical network analysis, electromagnetics, insulation design analysis in high voltage equipment and heat analysis in electrical and electronic equipment.
- It is used in chemical engineering include simulation of chemical processes, transport processes and chemical reaction simulations.
- It is used in meteorology include climate, monsoon and wind predictions.
- It is used in bioengineering include simulation of various human organs, blood circulation prediction and total synthesis of human body.

7. CONCLUSION

FDM is simple and easy to implement, but difficult to implement for irregular grid problems. FEM is more versatile and can be applied to most problems compared to the FDM and BEM. BEM has some specific advantages over FEM such as reduction in computational dimension and ease in handling of data. FDM is confined to the grid system and is not versatile compared to FEM or BEM. Hence, the best choice in many cases is the combination of these three methods in one or other form like FEM-BEM combination or BEM-FDM combination or FEM-FDM combination to suit any particular problem to the benefit, accuracy, efficient and economy. With advancements in computational science and development of modern computers and high speed processor, application of these grid methods has become much easier. In future, we are developing the algorithm based on these numerical grid techniques and which will be useful to other researcher.

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8. REFERENCES

[1] Zienkiewicz O. C., Finite Element Method, 3rd edition, McGraw, New York, 1980.

[2] Reddy J. N., An Introduction to the Finite Element Method, McGraw-Hill International, New York, 1985.

[3] Bathe K. J., Finite Element Procedure, Prentice Hall of India, New Delhi, 2001.

[4] Chapra S. C. and Canale R. P., Numerical Methods for Engineering, 4th edition, Tata McGraw-Hill, New Delhi, 2002.

[5] Srirekha A, Bashetty K. Infinite to finite: An overview of finite element analysis. Indian Journal of Dental Research, 2010, 21, 425-432.

[6] E. Nadal , J. J. Ródenas , J. Albelda , M. Tur , J. E. Tarancón , and F. J. Fuenmayor, Efficient Finite Element Methodology Based on Cartesian Grids: Application to Structural Shape Optimization, Hindawi : Abstract and Applied Analysis, 2013, 1-19.

[7] Neela Nataraj and A. S. Vasudeva Murthy, Finite element methods: Research in India over the last decade, Indian Journal of Pure and Applied Mathematics, 2019, 50, 739-765.

[8] Blanca Ayuso de Dios, Konstantin Lipnikov, and Gianmarco Manzini, The nonconforming virtual element method, SAIM Math. Model. Numer. Anal., 50(3) (2016), 879–904.

[9] S. Badia and R. Codina, Thirupathi Gudi, and J. Guzmán, Error analysis of discontinuous Galerkin methods for the Stokes problem under minimal regularity, IMA J. Numer. Anal., 34(2) (2014), 800–819.

[10] Saumya Bajpai and Neela Nataraj, On a two-grid finite element scheme combined with Crank-Nicolson method for the equations of motion arising in the Kelvin-Voigt model, Comput. Math. Appl., 68(12, part B) (2014), 2277–2291.

[11] Saumya Bajpai and Amiya K. Pani, On a three level two-grid finite element method for the 2D-transient Navier-Stokes equations, J. Numer. Math.(2016).

[12] Saumya Bajpai, Neela Nataraj, and A. K. Pani, On fully discrete finite element schemes for equations of motion of Kelvin-Voigt fluids, Int. J. Numer. Anal. Model., 10 (2013a), 481–507.

[13] Saumya Bajpai, Neela Nataraj, Amiya K. Pani, Pedro Damazio, and Jin Yun Yuan, Semidiscrete Galerkin method for equations of motion arising in Kelvin-Voigt model of viscoelastic fluid flow, Numer. Methods Partial Differential Equations, 29(3) (2013b), 857–883.

[14] Saumya Bajpai, Neela Nataraj, and Amiya K. Pani, On a two-grid finite element scheme for the equations of motion arising in Kelvin-Voigt model, Adv. Comput. Math., 40(5–6) (2014), 1043–1071.

[15] S. Balasuhdaram and P. K. Bhattacharyya, A mixed finite element method for fourth order elliptic equations with variable coefficients, Comput. Math. Appl., 10(3) (1984), 245–256

[16] S. Balasundaram and P. K. Bhattacharyya, A mixed finite element method for fourth order partial differential equations, Z. Angew. Math. Mech., 66(10) (1986), 489–499.

[17] S. Balasundaram and P. K. Bhattacharyya, A mixed finite-element method for fourth-order elliptic problems with variable coefficients, J. Comput. Appl. Math., 22(1) (1988), l–24.

[18] P. K. Bhattacharyya and S. Gopalsamy, An equilibrium finite element method for fourth order elliptic equations with variable coefficients, Comput. Math. Appl., 15(5) (1988), 405–424.

[19] Daniele Boffi, Franco Brezzi, and Michel Fortin, Mixed finite element methods and applications, 44, Springer Series in Computational Mathematics, Springer, Heidelberg, 2013.

[20] Andrea Bonito and Alan Demlow, Convergence and optimality of higher-order adaptive finite element methods for eigenvalue clusters, S1AM J. Numer. Anal., 54(4) (2016), 2379–2388.

[21] D. Braess, Finite elements, theory, fast solvers, and applications in elasticity theory, Cambridge, 3rd edition, 2007.

[22] Susanne C. Brenner, Two-level additive Schwarz preconditioners for nonconforming finite element methods, Math. Comp., 65(215) (1996), 897–921.

[23] Susanne C. Brenner and L. Ridgway Scott, The mathematical theory of finite element methods, 15, Texts in Applied Mathematics, Springer, New York, third edition, 2008.

[24] Susanne C. Brenner, Thirupathi Gudi, Michael Neilan, and Li-yeng Sung, Co penalty methods for the fully nonlinear Monge-Ampère equation, Math. Comp., 80(276) (2011), 1979–1995.

[25] Susanne C. Brenner, Shiyuan Gu, Thirupathi Gudi, and Li-yeng Sung, A quadratic Co interior penalty method for linear fourth order boundary value problems with boundary conditions of the Cahn-Hilliard type, SIAM J. Numer. Anal., 50(4) (2012), 2088–2110.

[26] Zhiqiang Cai, Cuiyu He, and Shun Zhang, Discontinuous finite element methods for interface problems: Robust a priori and a posteriori error estimates, SIAM J. Numer. Anal., 55(1) (2017), 400–418.

[27] Carsten Carstensen and M. Schedensack, Medius analysis and comparison results for first-order finite element methods in linear elasticity, IMA J. Numer. Anal., 35(4) (2015), 1591–1621.

[28] Carsten Carstensen, Gouranga Mallik, and Neela Nataraj, A priori and a posteriori error control of discontinuous Galerkin finite element methods for the Kármán equations, IMA J. Numer. Anal., 139(1) (2019), 167–200.

[29] Carsten Carstensen, D. Peterseim, and M. Schedensack, Comparison results of finite element methods for the Poisson model problem, SIAM J. Numer. Anal., 50(6) (2012), 2803–2823.

[30] Carsten Carstensen, Dietmar Gallistl, and Neela Nataraj, Comparison results of nonstandard P 2 finite element methods for the biharmonic problem, ESAIM Math. Model. Numer. Anal., 49(4) (2015), 977–990.

[31] Carsten Carstensen, Asha K. Dond, Neela Nataraj, and Amiya K. Pani, Error analysis of nonconforming and mixed FEMs for second-order linear non-selfadjoint and indefinite elliptic problems, Numer. Math., 133(3) (2016a), 557–597.
[32] Carsten Carstensen, Neela Nataraj, and Amiya K. Pani, Comparison results and unified analysis for firstorder finite volume element methods for a Poisson model problem, IMA J. Numer. Anal., 36(3) (2016b), 1120–1142.
[33] Sudipto Chowdhury, Thirupathi Gudi, and A. K. Nandakumaran, Error bounds for a Dirichlet boundary control problem based on energy spaces, Math. Comp., 86(305) (2017), 1103–1126.
[34] P. G. Ciarlet, The finite element method for elliptic problems, North-Holland, Amsterdam, 1978.
[35] P. G. Ciarlet, Liliana Gratie, and Srinivasan Kesavan, Numerical analysis of the generalized von Kármán equations, C. R. Math. Acad. Sci. Paris, 341(11) (2005), 695–699.
[36] D. A. Di Pietro and A. Ern, Mathematical aspects of discontinuous Galerkin methods, Mathematiques et Applications, Springer Berlin Heidelberg, 2011.
[37] Asha K. Dond and Amiya K. Pani, A priori and a posteriori estimates of conforming and mixed FEM for a Kirchhoff equation of elliptic type, Comput. Methods Appl. Math., 17 (2) (2017), 217–236.
[38] Asha K. Dond, Thirupathi Gudi, and Neela Nataraj, On a nonconforming finite element approximation for optimal control of the obstacle problem, Comput. Methods Appl. Math., 16(4) (2016), 653–666.
[39] Asha K. Dond, Neela Nataraj, and Amiya Kumar Pani, Convergence of an adaptive lowest-order Raviart-Thomas element method for general second-order linear elliptic problems, IMA J. Numer. Anal., 37(2) (2017), 832–860.
[40] Jerome Droniou, Neela Nataraj, and Devika Shailaja, Numerical analysis for the pure Neumann control problem using the gradient discretisation method, Comput. Methods Appl. Math., 18(4) (2018), 609–637.
[41] S. Gaddam and Thirupathi Gudi, Bubbles enriched quadratic finite element method for the 3d-elliptic obstacle problem, Comput. Meth. Appl. Math., 18(2) (2018), 223–236.
[42] Dietmar Gallistl, Morley finite element method for the eigenvalues of the biharmonic operator, IMA J. Numer. Anal., 35(4) (2015), 1779–1811.
[43] S. Ganesan and S. Srivastava, ALE-SUPG finite element method for convection-diffusion problems in time-dependent domains: Conservative form, Appl. Math. Comp., 303 (2017), 128–145.
[44] S. Ganesan and L. Tobiska, Finite elements: Theory and algorithms, Cambridge IISc Series, Cambridge University Press, 2017.
[45] Vivette Girault, Jizhou Li, and Beatrice Rivière, Strong convergence of discrete DG solutions of the heat equation, J. Numer. Math., 24(4) (2016), 235–252.
[46] Deepjyoti Goswami, Amiya K. Pani, and Sangita Yadav, Optimal error estimates of two mixed finite element methods for parabolic integro differential equations with nonsmooth initial data, J. Sci. Comput., 56 (2013), 131–164.
[47] Thirupathi Gudi, A new error analysis for discontinuous finite element methods for linear elliptic problems, Math. Comp., 79(272) (2010a), 2169–2189.
[48] Thirupathi Gudi, Some nonstandard error analysis of discontinuous Galerkin methods for elliptic problems, Calcolo, 47(4) (2010b), 239–261.
[49] Thirupathi Gudi, Finite element method for a nonlocal problem of Kirchhoff type, SIAM J. Numer. Anal., 50(2) (2012), 657–668.
[50] M. K. Kadalbajoo and A. S. Yadaw, Finite difference, finite element and B-spline collocation methods applied to two parameter singularly perturbed boundary value problems, JNAIAM. J. Numer. Anal. Ind. Appl. Math., 5(3–4) (2011), 163–180.