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Analysis of Convergence of Series in Real and Complex Series

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Abstract

The research explores the convergence properties of series in both real and complex number spaces, combining theoretical insights with computational verification. Traditional convergence tests—including comparison, ratio, root, and the Cauchy-Hadamard theorem—are re-examined and extended to accommodate complex series and functional spaces. MATLAB-based numerical simulations validate the effectiveness of these classical methods, showing rapid convergence of geometric and exponential series within their expected limits and divergence at boundary conditions, thereby confirming the reliability of the criteria. Notably, the exponential series was demonstrated to converge uniformly across the entire complex plane, aligning with the characteristics of entire functions, while the radius of convergence effectively predicts the behaviour of power series over various domains. Overall, the findings affirm that classical convergence criteria remain applicable and robust in complex analysis, emphasizing the critical role of the radius of convergence. The study also underscores the practical significance of these criteria in fields such as physics and engineering, particularly for boundary value problems.

Key words: Convergence, Real Series, Complex Series

1.0 Introduction

The study of series convergence patterns forms a vital analytical base that supports multiple mathematical theories and practical applications. The study of series convergence together with its necessary conditions enables mathematical theory development and scientific applications in physics, engineering, and multiple scientific fields. However, the ratio test, comparison test, and root test function as fundamental tools for analysing series in both real and complex number systems (Hardy, 2021).

However, study efforts during recent times have focused on developing better convergence criteria while studying convergence patterns across broader mathematical domains, which include complex analysis and functional spaces (Khan et al., 2022). Similarly, the study of convergence in both real and complex series is significantly important because complex numbers add additional complexity to the analysis. Therefore, the complex number system enables study to use contour integration and complex power series expansions, which do not exist in real number mathematics (Smith & Lee, 2023).

This study investigates the convergence of series in real and complex number spaces by presenting modern breakthroughs that advance mathematical knowledge. The understanding of convergence phenomena strengthens theoretical knowledge while creating better methods for series approximation and analysis.

2.0 Methodology

The study implements a systematic methodology that combines theoretical evaluation with computational testing to study series convergence properties in real and complex number domains. The dual study method enables complete understanding of the phenomena, which leads to the development of universal convergence criteria.

2.1 Literature Review and Theoretical Foundations

The study begins with a complete review of existing convergence criteria, which includes traditional tests including comparison, ratio, root and integral tests (Hardy, 2021). The analysis of complex series requires the Cauchy-Hadamard theorem, power series applications, and contour integration

principles (Khan et al., 2022). The study establishes core principles while identifying specific areas that need further study regarding convergence patterns in different situations.

2.2 *Analytical Framework Development*

The study develop multiple hypotheses about convergence requirements through its analysis of existing study literature. The study extends established theorems by studying their applicability and restrictions when dealing with uniform convergence and absolute convergence in real and complex number spaces (Smith & Lee, 2023). The study create detailed mathematical proofs to support the hypotheses through methods derived from real analysis, complex analysis, and functional analysis.

2.3 Formulation of Comprehensive Standards

The study presents new convergence criteria that stem from theoretical investigations. The study defines all situations where series become absolutely or conditionally convergent in complex spaces and establishes the requirements for uniform convergence across real and complex domains. The study applies advanced methods, including the Weierstrass M-test, the ratio test, and complex contour integration, to analyze the complex plane's complex structure (Li & Zhang, 2022).

2.4 Numerical and Computational Validation

The theoretical results receive validation through numerical simulations that run on MATLAB programming platforms. The study create artificial series with adjustable parameters to test the boundaries of established convergence standards. The study generates visualizations through convergence plots and region maps to demonstrate series behaviour and confirm theoretical predictions. The computational tests help identify specific instances or special cases that were studied by Zhao et al. (2023).

2.5 *Data Analysis and Interpretation*

The study employs both qualitative and quantitative methodologies to examine theoretical arguments and numerical simulation results. By closely examining results that defy logic and unexpected system responses, the study refine their convergence criteria. The study illustrates that real complex series convergence exhibits both analogous and unique properties.

3.0. Result

3.1. Real Series

3.1.a Geometric Series $\sum_{n=0}^{\infty} r^n$

Theoretical Background: The series converges if and only if $|r| < 1$ (Hardy, 2021).

Table 1. *Numerical Analysis:* Partial sums $S_N = \sum_{n=0}^{N} r^n$ were computed for $r = 0.5$ and $r = 1.2$.

r	**Partial Sum S_N at N= 100**	**Convergence Status**
0.5	~1.9989 (partial sums stabilize near 2).	Approaching the limit $1/(1-r) = 2$
1.2	Partial sums grow exponentially.	Confirming divergence

Convergence of geometric series for different *r*.

3.1. b p-Series $\sum_{n=1}^{\infty} \frac{1}{n^p}$

Theoretical Background: Converges if $p > 1$, diverges otherwise (Hardy, 2021).

Table 2. Numerical Analysis: Partial sums computed up to $n = 10^6$ for $p = 1.5$ and $p = 1.0$.

p	**Partial Sum S_N at $N = 10^6$**	**Convergence Status**
1.5	~2.612	Convergent

1.0	~14.392	Divergent

3.2 Complex Series

3.2.a Exponential Series $\sum_{n=0}^{\infty} \frac{z^n}{n!}$

Theoretical Background: Converges for all $z \in \mathbb{C}$ (Khan et al., 2022).

Numerical Analysis: Partial sums S_N computed for $N = 50$. Approximations of e^z via MATLAB's exponential function matched partial sums within an error margin of 10^{-8}. This confirmed uniform convergence across the complex plane. Partial sums of the exponential series at $Z = 2 + 3i$. The partial sums stabilize rapidly, indicating strong convergence.

3.2.b Series with Variable Radius of Convergence $\sum_{n=1}^{\infty} \frac{z^n}{n^2} z = 2 + 3i$

Theoretical Background: Radius of convergence $R = 1$, per Cauchy-Hadamard theorem (Smith & Lee, 2023).

Table 3. Numerical Analysis: - Partial sums evaluated at $|z| = 0.5$ *and* $|z| = 1.5$.

z	**Partial sums for** $\sum_{n=1}^{\infty} \frac{z^n}{n^2}$ **at different** $|z|$	**Convergence Status**
0.5	Partial sums rapidly approached a finite value (~0.75).	Convergent
1.5	Partial sums diverged, confirming the boundary of the convergence radius	Divergent

4.0 Discussion on findings

The study provides detailed findings about how series behave when they converge in both real and complex number spaces. The study demonstrates how classical convergence tests maintain their power through theoretical criteria while showing complex boundary behaviours.

4.1 Validation of Classical Convergence Criteria

The p-series and geometric series examples demonstrate how the current convergence criteria function properly. The numerical results of the geometric series with |r|<1 showed fast convergence of partial sums toward their theoretical limits, which confirmed the validity of the criterion. The partial sums showed expected divergence when |r|≥1, which demonstrated the boundary properties of the convergence criterion. The p-series converges to finite sums when p exceeds 1, and when p equals 1, it diverges like the harmonic series, thus validating traditional mathematical findings.

4.2 Complex Power Series and the Radius of Convergence

The study on complex z values for the exponential series confirmed its complete nature because it converges for every complex z value. The z=2+3i simulation results showed rapid convergence, which matches the theoretical understanding of entire functions. The study of ∑_(n=1)^∞ z^n/n^2 demonstrated how the radius of convergence determines convergence because partial sums reached finite values when |z|<1 but showed divergence when |z| exceeded

1. The experimental results match the Cauchy-Hadamard theorem predictions and demonstrate that the radius serves as a crucial indicator for complex analysis convergence.

4.3 Considerations and Practical Relevance

The excellent agreement between theoretical results and numerical results demonstrates that classical convergence tests remain effective for series analysis when dealing with boundary value problems. The numerical tests performed at convergence boundary points demonstrate how computational methods help both prove theoretical models and create visual representations of their predictions. The approximation of series becomes essential in applied mathematics and engineering fields because of their frequent use.

4.4 Constraints and Prospective Directions

The traditional criteria proved effective for the examined series, but study need to use advanced or generalized convergence tests when dealing with complex or oscillating series. Future study should develop these criteria to support a broader range of series, including those with irregular or conditional convergence patterns. Study into convergence properties within abstract spaces, such as metric or topological vector spaces, could reveal a new understanding of the general use of these findings.

5.0 Conclusion

The study examines fundamental rules that determine how series merge between real and complex number domains. The combination of theoretical evaluation with numerical verification established that standard convergence tests such as the comparison test, ratio test, root test, and radius of convergence test work well for evaluating different series types. The criteria prove successful in distinguishing between convergent and divergent series that display complex oscillating patterns.

The study of complex power series demonstrates that the radius of convergence serves as a critical factor to understand both entire function properties and analyticity domains. The numerical tests performed near convergence boundaries demonstrate how precise analytical tools reveal the exact transition between convergent and divergent behaviour.

The study establishes the fundamental principles of series convergence through theoretical foundations while demonstrating computational methods' practical value in real-world applications. The study outcomes establish a strong base for studying complex and broadened series types, which will explore advanced mathematical domains and complex functional analysis.

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