JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

RAHMOH AND SHEHU TRANSFORMS FOR SOLVING SOME NONLINEAR INTEGRAL EQUATIONS

[1]Manpreet Kaur, [2]Inderdeep Singh
[1]PG Student, [2]Associate Professor of Mathematics
[1,2] Department of Physical Sciences
[1,2] Sant Baba Bhag Singh University, Jalandhar-144030, Punjab, India
Email: manpreetkaur191512@gmail.com, inderdeeps.ma.12@gmail.com,

Abstract
In this research, two efficient semi-analytical techniques have been developed for solving nonlinear integral equations, which frequently arise in various fields of applied mathematics, physics, and engineering. The proposed approaches are based on the Rahmoh transform and the Shehu transform, both of which provide powerful frameworks for handling nonlinear integral equations with improved computational efficiency. The main objective of this study is to construct reliable and accurate solution methodologies that overcome the limitations of existing classical and numerical techniques. The developed schemes are simple to implement and reduce computational complexity while maintaining high accuracy.

Keywords
Rahmoh transform; Shehu transform; Nonlinear Integral Equations, Numerical Experiments.

I. INTRODUCTION

Nonlinear integral equations arise in many areas of applied mathematics, physics, and engineering, where the unknown function appears in a nonlinear form under the integral sign. Unlike linear integral equations, these equations are generally more complex and often do not admit closed-form solutions. They play a significant role in modeling real-world phenomena such as population dynamics, fluid flow, heat transfer, and biological systems. Due to their complexity, various analytical and numerical techniques such as iterative methods, decomposition methods, and integral transforms are commonly employed to obtain approximate or exact solutions.

In [1], the authors applied the Shehu transform to solve Volterra integral equations of the first kind, which commonly arise in areas such as thermodynamics, reactor theory, and electrical systems. The study includes numerical examples to illustrate the applicability of the method. The results demonstrate that the Shehu transform is an effective and reliable tool for handling such equations. In [2], the authors applied the Shehu transform to model and solve growth and decay problems arising in various scientific fields. The study presents numerical examples to demonstrate the effectiveness of the method. The results indicate that the Shehu transform is a useful and efficient tool for analyzing such problems. In [3], the authors introduced the Shehu transform, a generalized Laplace-type integral transform that unifies the properties of the Laplace and Sumudu transforms. It is formulated based on the classical Fourier integral transform and is effectively employed in solving both ordinary and partial differential equations, highlighting its simplicity and computational effectiveness. In [4], the authors applied the Shehu transform to solve Abel's integral equation, a significant singular integral equation arising in various scientific fields. The study includes several numerical examples to illustrate the effectiveness and accuracy of the proposed approach. In [5], the authors established the effectiveness of the Shehu transform for fractional Riemann–Liouville and Caputo operators. The method is applied to solve higher-order fractional differential equations in the Caputo sense, supported by illustrative examples to demonstrate its applicability. In [6], the authors improved the computational framework of the Shehu transform by introducing an explicit inversion formula. The study further extends the transform to generalized settings, including distributions and measures, thereby enhancing its theoretical scope and applicability. In [7], the authors applied the Shehu transform to solve ordinary differential equations with constant coefficients. The method is further used in applications such as electric circuits, damped and undamped motion, and Newton's law of cooling, demonstrating its practical relevance in physics. In [8], the authors discussed the fundamental properties of the Shehu transform and its application in solving differential equations. The study applies the transform to both ordinary and partial differential equations and further extends its use to transport and heat equations under various initial conditions, demonstrating its effectiveness. In [9], the authors introduced the Shehu transform, proposed by Maitama, as a generalization of

the Laplace and Sumudu transforms. The study establishes key results, including the convolution and uniqueness theorems. These results are further applied to solve certain integral equations, demonstrating the usefulness of the transform. In [10], the authors investigated the time-fractional Klein–Gordon equation using the natural transform decomposition method and the iterative Shehu transform method with a singular kernel. The solution is obtained by applying the transforms and their inverses. Numerical results are presented to validate the accuracy and efficiency of the methods. The study concludes that these approaches are reliable for analyzing fractional differential equations. In [11], the authors applied the Shehu transform to solve important partial differential equations. General solution formulas are derived for Laplace, Poisson, and Telegraph equations under given initial and boundary conditions. The study demonstrates the effectiveness of the method in handling such problems. In [12], the authors proposed a fractional Shehu transform of order $0 < \alpha < 1$, for functions with fractional differentiability. The study establishes its fundamental properties based on the new definition. Additionally, convolution and inversion results are derived, highlighting the theoretical development of the transform. In [13], the authors introduced an iterative Shehu transform method for solving time-fractional hyperbolic telegraph equations in multiple dimensions. The approach is applied to problems in physics, and its accuracy is validated through comparisons of approximate and exact solutions using graphical analysis. The method is shown to be efficient and reliable for solving ODEs, PDEs, and fractional equations, supported by a convergence study. In [14], the authors developed the Shehu transform for solving both homogeneous and non-homogeneous linear differential equations. The method is applied to obtain solutions of higher-order linear differential equations in an efficient manner. The study also introduces a new approach to examine Hyers–Ulam stability using the Shehu transform. Furthermore, applications to fractional differential equations, Newton's law of cooling, and free undamped motion are included to demonstrate its effectiveness. In [15], the authors introduced a semi-analytical approach, namely the variational iteration transformation method, for solving nonlinear homogeneous and nonhomogeneous fractional-order gas dynamics equations. The method combines the Shehu transform with an iterative scheme to obtain solutions efficiently. Its performance is validated through several test problems, showing improved accuracy and simplicity without requiring complex computations. The study concludes that the method is effective and can be extended to a wide range of nonlinear fractional problems. In [16], the authors introduced the double Shehu transform as a generalization of the double Laplace and double Sumudu transforms. The study derives its formulation for fractional Caputo operators and applies it to solve fractional partial differential equations involving Caputo derivatives, demonstrating its effectiveness. In [17], the authors proposed the Shehu Transform Adomian Decomposition Method (STADM) for solving both linear and nonlinear integral as well as integro-differential equations. The method is applied to initial value problems with constant and variable coefficients, utilizing the generality of the Shehu transform over Laplace and Sumudu transforms. Nonlinear terms are handled using Adomian polynomials. The effectiveness of the approach is demonstrated through selected examples, yielding accurate and reliable results. In [18], the authors introduced the Shehu transform in the context of generalized functions. The study establishes an inversion theorem along with uniqueness and characterization results, highlighting the theoretical development of the transform. In [19], the authors introduced a novel double hybrid transform that unifies and generalizes the Hybrid Sumudu and Shehu transforms into a single integral framework. The study established its fundamental properties, including existence, inversion, and related theorems. It also extended results for partial derivatives and generalized the double convolution theorem. Furthermore, the transform was applied to solve certain classes of differential equations, demonstrating its efficiency and applicability in scientific problems. In [20], the authors employed the Shehu transform to solve linear Volterra integral equations and second-order integro-differential equations. The method is illustrated through several numerical examples to demonstrate its applicability. The results show that it provides exact solutions with reduced computational effort, making it an efficient and accurate alternative to traditional methods. In [21], the authors introduced the RAHMOH transform, a Laplace-type integral transform that generalizes both the Laplace and Sumudu transforms for solving ordinary and partial differential equations. The study establishes its existence, inverse formulation, and fundamental properties, along with relevant theorems. Applications are also provided to demonstrate its effectiveness in solving differential equations. In [22], the authors presented a comparative study of several integral transforms, including Laplace, Sumudu, Elzaki, Aboodh, and the RAHMOH transform. A unified framework is developed to examine their kernel structures, and key properties of the RAHMOH transform, including its inversion and equivalence to the Laplace transform, are established. The methods are applied to solve ODEs, PDEs, and fractional differential equations, highlighting differences in their analytical procedures. Numerical results further confirm the accuracy and stability of the RAHMOH transform in various applications. In [23], the authors proposed hybrid numerical methods for solving Volterra integral equations of the first kind. The equations are first transformed into second kind form using Leibniz rule, and then solved using the Adomian decomposition and variational iteration methods. Numerical examples are provided to demonstrate the accuracy and effectiveness of the approach. In [24], the authors studied the two-dimensional time-fractional Rosenau–Hyman equation using the Elzaki transform combined with the homotopy perturbation method. The study includes convergence analysis and numerical examples to demonstrate the simplicity and accuracy of the proposed approach. In [25], the authors studied solitary wave solutions of two- and three-dimensional time-fractional nonlinear Rosenau–Hyman equations. The method combines the Sumudu transform with the homotopy perturbation technique, and convergence results are established. Several test examples are provided to demonstrate the accuracy and effectiveness of the approach. In [26], the authors investigated two- and three-dimensional telegraph equations using a combined approach based on the Elzaki transform and the homotopy perturbation method. In [27], the authors proposed a technique by combining the Elzaki transform with the homotopy perturbation technique to obtain analytical solutions of two-dimensional wave propagation equations. Numerical experiments are presented to demonstrate the accuracy and simplicity of the approach, highlighting its usefulness in scientific and engineering applications.

This paper is constituted as: Section Ⅰ contain the introductory part of the paper. Rahmoh transform has been discussed in Section Ⅱ.Shehu transform has been explained in Section Ⅲ. In Section Ⅳ, proposed methodology has been discussed. Numerical examples have been solved to illustrate the simplicity and accuracy of the proposed schemes in Section Ⅴ. Results and discussion have been presented in Section Ⅵ. Conclusion has been discussed in Section Ⅶ.

II. RAHMOH TRANSFORM

The Rahmoh transform is a recently proposed integral transform developed by Rahma A. Farah and Mohamed A. Hamad(2024), as a new Laplace-type method for solving differential equations. It is designed to simplify the analytical treatment of mathematical models that arise in applied sciences.

Definition 1: Let $\mathfrak{f}(t)$ be a function of exponential order defined on $[0,\infty)$.The Rahmoh transform of $\mathfrak{f}(t)$,denoted by $\mathcal{M}\{\mathfrak{f}(t)\}$, is defined as:

$$\mathcal{M}\{\mathfrak{f}(t)\}(s,u)=\mathcal{M}(s,u)=u^2\int_0^{\infty}e^{-\frac{s}{u}t}\,\mathfrak{f}(t)dt,\quad s>0,u>0$$

The transform uses the kernel

$$\mathcal{K}_M(s,u,t)=u^2e^{-\frac{s}{u}t}$$

This transform depends on two parameters s and u, which makes it a generalized form of classical integral transforms.

Inverse Rahmoh Transform

The inverse of the Rahmoh transform is defined through a complex contour integral. If $\mathcal{M}(s,u)$ is the Rahmoh transform of $\mathfrak{f}(t)$, then the original function can be recovered by:

$$\mathfrak{f}(t)=\mathcal{M}^{-1}\{\mathcal{M}(s,u)\}=\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\frac{1}{u^2}\mathcal{M}(s,u)e^{\frac{s}{u}t}\,d\left(\frac{s}{u}\right)$$

where γ is a real constant chosen such that all singularities lie to the left of the contour in the complex plane.

Basic Condition (Existence)

The Rahmoh transform exists if the integral converges, i.e., if $\mathfrak{f}(t)$ is of exponential order such that:

$$|\mathfrak{f}(t)|\leq\mathcal{M}e^{kt},\quad t\geq 0$$

For some constants $\mathcal{M}>0$ and k.

III. SHEHU TRANSFORM

The Shehu transform is a modern integral transform technique developed to solve differential and integral equations arising in science and engineering. It extends the idea of classical transforms such as the Laplace and Fourier transforms by introducing an additional parameter, which enhances flexibility in handling a wider class of problems.

Definition 2: For a function $v(t)$of exponential order, the Shehu transform is defined as:

$$\mathbb{S}\{v(t)\}=\mathcal{V}(s,u)=\int_0^{\infty}e^{-\frac{st}{u}}v(t)\,dt,\ s>0,\ u>0$$

where s and u are the transform variables.

Inverse Shehu Transform

The inverse transform is used to recover the original function and is given by:

$$\mathbb{S}^{-1}\{\mathcal{V}(s,u)\}=v(t)$$

This allows transformation of algebraic solutions back into the time domain.

Existence Conditions

The Shehu transform exists if:

The function $v(t)$ is piecewise continuous on every finite interval. The function is of exponential order, *i.e.*,

$$|\,v(t)\,|\leq\mathcal{N}e^{\alpha t},\ t>\beta$$

for some constants $\mathcal{N},\alpha,\beta$.

Table 1.: Comparison table of rahmoh transform & shehu transform

$\mathfrak{f}(t)$	$\mathcal{M}\{\mathfrak{f}(t)\}$	$\mathbb{S}\{\mathfrak{f}(t)\}$
t^n	$\frac{n!\,u^{n+3}}{s^{n+1}}$	$\frac{n!\,u^{n+1}}{s^{n+1}}$

e^{at}	$\frac{u^3}{s-au}$	$\frac{u}{s-au}$
$sin(at)$	$\frac{au^4}{s^2+a^2u^2}$	$\frac{au^2}{s^2+a^2u^2}$
$\cos(at)$	$\frac{su^3}{s^2+a^2u^2}$	$\frac{su}{s^2+a^2u^2}$
$\sin h\,(at)$	$\frac{au^4}{s^2-a^2u^2}$	$\frac{au^2}{s^2-a^2u^2}$
$\cos h\,(at)$	$\frac{su^3}{s^2-a^2u^2}$	$\frac{su}{s^2-a^2u^2}$
t^{α}	$\frac{\Gamma(\alpha+1)u^{\alpha+3}}{s^{\alpha+1}}$	$\frac{\Gamma(\alpha+1)u^{\alpha+1}}{s^{\alpha+1}}$

IV. PROPOSED METHODOLOGY

In this section, we present some lemmas and a proposed scheme for solving nonlinear integral equations of the first kind.

Lemma 1: If $\mathcal{M}\{\mathcal{V}(x)\} = \bar{\mathcal{V}}(s,u)$, then $\mathcal{M}\{\int_0^x \mathcal{V}(t)\,dt\} = \frac{u}{s}\mathcal{M}\{\mathcal{V}(x)\}$

Let

$$\phi(x) = \int_0^x \mathcal{V}(t)dt$$

This implies

$$\phi'(x) = \mathcal{V}(x), \quad \phi(0) = 0$$

Applying Rahmoh transform, we obtain

$$\mathcal{M}\{\phi'(x)\} = \frac{s}{u}\mathcal{M}\{\phi(x)\} - u^2\phi(0) = \mathcal{M}\{\mathcal{V}(x)\}$$

Using initial conditions, we obtain

$$\mathcal{M}\{\phi'(x)\} = \frac{s}{u}\mathcal{M}\{\phi(x)\} = \mathcal{M}\{\mathcal{V}(x)\}$$

This implies

$$\mathcal{M}\{\phi(x)\} = \frac{u}{s}\mathcal{M}\{\mathcal{V}(x)\}$$

i.e.

$$\mathcal{M}\left\{\int_0^x \mathcal{V}(t)\,dt\right\} = \frac{u}{s}\mathcal{M}\{\mathcal{V}(x)\}$$

Lemma 2: If $\mathbb{S}\{\mathcal{V}(x)\} = \bar{\mathcal{V}}(s,u)$, then $\mathbb{S}\{\int_0^x \mathcal{V}(t)\,dt\} = \frac{u}{s}\mathbb{S}\{\mathcal{V}(x)\}$

Let

$$\phi(x) = \int_0^x \mathcal{V}(t)dt$$

This implies

$$\phi'(x) = \mathcal{V}(x), \qquad \phi(0) = 0$$

Applying Shehu transform, we obtain

$$\mathbb{S}\{\phi'(x)\} = \frac{s}{u}\mathbb{S}\{\phi(x)\} - \phi(0) = \mathbb{S}\{\mathcal{V}(x)\}$$

Using initial condition, we obtain

$$\mathbb{S}\{\phi'(x)\} = \frac{s}{u}\mathbb{S}\{\phi(x)\} = \mathbb{S}\{\mathcal{V}(x)\}$$

This implies

$$\mathbb{S}\{\phi(x)\} = \frac{u}{s}\mathbb{S}\{\mathcal{V}(x)\}$$

This implies

$$. \mathbb{S}\{\int_0^x \mathcal{V}(t)\,dt\} = \frac{u}{s}\mathbb{S}\{\mathcal{V}(x)\}$$

Proposed Scheme for Solving Nonlinear Integral Equations:

Consider the general nonlinear integral equation of the first kind:

$$\int_0^x \mathcal{N}\{u(t)\}\,dt = \mathfrak{f}(x) \qquad (1)$$

Substituting $\mathcal{N}\{u(x)\} = \mathcal{V}(x)$in the equation (1),we get

$$\int_0^x \mathcal{V}(t)\,dt = \mathfrak{f}(x) \tag{2}$$

(a) Rahmoh Transform Based Technique

Applying the Rahmoh transform on the both sides of the equation(2), we obtain

$$\mathcal{M}\left\{\int_0^x \mathcal{V}(t)\,dt\right\} = \mathcal{M}\{\mathfrak{f}(x)\}$$

Using above Lemma 1, we obtain

$$\frac{u}{s}\mathcal{M}\{\mathcal{V}(x)\} = \bar{\mathfrak{f}}(s,u) \tag{3}$$

Divide both sides of equation (3)by $\frac{u}{s}$, to isolate the transform of the solution.

$$\mathcal{M}\{\mathcal{V}(x)\} = \frac{\bar{\mathfrak{f}}(s,u)}{\frac{u}{s}}$$

This implies

$$\mathcal{M}\{\mathcal{V}(x)\} = \frac{s.\bar{\mathfrak{f}}(s,u)}{u} \tag{4}$$

Applying inverse Rahmoh transform on the both sides of the equation(4), we obtain

$$\mathcal{V}(x) = \mathcal{M}^{-1}\left\{\frac{s.\bar{\mathfrak{f}}(s,u)}{u}\right\}$$

After simplifications, we obtained the required solution.

(b) Shehu Transform Based Technique

Taking Shehu transform of both sides of equation(2), we obtain

$$\mathbb{S}\left\{\int_0^x \mathcal{V}(t)\,dt\right\} = \mathbb{S}\{\mathfrak{f}(x)\}$$

Using above Lemma 2, we obtain

$$\frac{u}{s}\mathbb{S}\{\mathcal{V}(x)\} = \mathcal{F}(s,u) \tag{5}$$

Now, divide equation(5) by $\frac{u}{s}$,to isolate the transform of the solution.

$$\mathbb{S}\{\mathcal{V}(x)\} = \frac{\mathcal{F}(s,u)}{\frac{u}{s}}$$

The equation becomes

$$\mathbb{S}\{\mathcal{V}(x)\} = \frac{s.\mathcal{F}(s,u)}{u} \tag{6}$$

Applying inverse Shehu transform on the both sides of the equation(6), we obtain

$$\mathcal{V}(x) = \mathbb{S}^{-1}\left\{\frac{s.\mathcal{F}(s,u)}{u}\right\}$$

After simplification, we obtain the required solution.

V. NUMERICAL EXPERIMENTS

In this Section, we present some numerical examples to illustrate the simplicity and accuracy of the proposed schemes.

Example 1: Solve the given equation

$$\int_0^x u^2(t)dt = \frac{x}{2} - \frac{sin2x}{4} \tag{7}$$

The given integral equation is:

$$\int_0^x u^2(t)dt = \frac{x}{2} - \frac{sin2x}{4}$$

Substituting $u^2(x) = \mathcal{V}(x)$ in the above equation, we obtain

$$\int_0^x \mathcal{V}(t)dt = \frac{x}{2} - \frac{sin2x}{4} \tag{8}$$

(a) Rahmoh Transform Method Based Technique

Applying the Rahmoh transform on both sides of the equation(8), we obtain

$$\mathcal{M}\left\{\int_0^x \mathcal{V}(t)dt\right\} = \mathcal{M}\left\{\frac{x}{2} - \frac{sin2x}{4}\right\}$$

Using above Lemma 1, we obtain

$$\frac{u}{s}\mathcal{M}\{\mathcal{V}(x)\} = \frac{1}{2}\left\{\frac{u^4}{s^2}\right\} - \frac{1}{4}\left\{\frac{2u^4}{s^2 + 2^2u^2}\right\}$$

Multiply both side by $\frac{s}{u}$, we obtain

$$\mathcal{M}\{\mathcal{V}(x)\} = \frac{1}{2}\left\{\frac{u^3}{s}\right\} - \frac{1}{2}\left\{\frac{su^3}{s^2 + 4u^2}\right\} \quad (9)$$

Applying the inverse Rahmoh transform on the both sides of equation(9), we obtain

$$\mathcal{V}(x) = \frac{1}{2}\mathcal{M}^{-1}\left\{\frac{u^3}{s}\right\} - \frac{1}{2}\mathcal{M}^{-1}\left\{\frac{su^3}{s^2 + 4u^2}\right\}$$

Using the standard Inverse Rahmoh transform formula, we obtain

$$\mathcal{V}(x) = \frac{1}{2}(1 - cos2x) = sin^2x$$

To solve for $u(x)$, we substitute the value of $\mathcal{V}(x)$ into the expression:

$$u^2(x) = \mathcal{V}(x) = sin^2x$$

This implies

$$u(x) = sinx \quad (10)$$

which is an exact solution of equation(7).

(b) **Shehu Transform Method Based Technique**

Applying the Shehu transform on the both sides of the equation(8), we obtain

$$\mathbb{S}\left\{\int_0^x \mathcal{V}(t)dt\right\} = \mathbb{S}\left\{\frac{x}{2} - \frac{sin2x}{4}\right\}$$

Using above Lemma 2, we obtain

$$\frac{u}{s}\mathbb{S}\{\mathcal{V}(x)\} = \frac{1}{2}\left\{\frac{u^2}{s^2}\right\} - \frac{1}{4}\left\{\frac{2u^2}{s^2 + 2^2u^2}\right\}$$

Multiply the both sides by $\frac{s}{u}$, we obtain

$$\mathbb{S}\{\mathcal{V}(x)\} = \frac{u}{2s} - \frac{su}{2(s^2 + 4u^2)} \quad (11)$$

Using the inverse Shehu transform of each term of the equation (11), we obtain

$$\mathcal{V}(x) = \frac{1}{2}\mathbb{S}^{-1}\left\{\frac{u}{s}\right\} - \frac{1}{2}\mathbb{S}^{-1}\left\{\frac{su}{s^2 + 4u^2}\right\}$$

This implies

$$\mathcal{V}(x) = \frac{1 - cos2x}{2} = sin^2x$$

Substitute the value of $\mathcal{V}(x)$ into the expression:

$$u^2(x) = \mathcal{V}(x) = sin^2x \quad (12)$$

This implies

$$u(x) = sinx$$

Example 2. Solve the given equation

$$\int_0^x sin\, u(t)dt = \frac{x^2}{2} \quad (13)$$

Substituting $\sin u(x) = \mathcal{V}(x)$ into the above given equation

The equation becomes:

$$\int_0^x \mathcal{V}(t)dt = \frac{x^2}{2} \quad (14)$$

(a) **Rahmoh Transform Method Based Technique**

Applying Rahmoh transform on the both sides of the equation(14), we obtain

$$\mathcal{M}\left\{\int_0^x \mathcal{V}(t)dt\right\} = \frac{1}{2}\mathcal{M}\{x^2\}$$

Using the above Lemma 1, we obtain

$$\frac{u}{s}\mathcal{M}\{\mathcal{V}(x)\} = \frac{1}{2}.2\left\{\frac{u^5}{s^3}\right\}$$

Divide both sides by $\frac{s}{u}$ to solve for the Rahmoh transform, we obtain

$$\mathcal{M}\{\mathcal{V}(x)\} = \frac{u^4}{s^2} \quad (15)$$

Now, we take the inverse Rahmoh transform of the both sides of the equation(15), we obtain

$$\mathcal{V}(x) = \mathcal{M}^{-1}\left\{\frac{u^4}{s^2}\right\}$$

This implies

$$\mathcal{V}(x) = x$$

Substitute $\mathcal{V}(x)$ back to find $u(x)$:

$$\sin u(x) = \mathcal{V}(x) = x \tag{16}$$

This implies

$$u(x) = sin^{-1}x$$

This is an exact solution.

(b) **Shehu Transform Method Based Technique**

Applying the Shehu transform to the equation(14), we obtain

$$\mathbb{S}\left\{\int_0^x \mathcal{V}(t)dt\right\} = \frac{1}{2}.\mathbb{S}\{x^2\}$$

Using the above Lemma 2, we obtain

$$\frac{u}{s}.\mathbb{S}\{\mathcal{V}(x)\} = \frac{1}{2}.\mathbb{S}\{x^2\}$$

Using the standard transform for x^n,

$$\frac{u}{s}.\mathbb{S}\{\mathcal{V}(x)\} = \frac{1}{2}.\left\{2\frac{u^3}{s^3}\right\}$$

After simplification, we get

$$\mathbb{S}\{\mathcal{V}(x)\} = \frac{u^2}{s^2} \tag{17}$$

To find $\mathcal{V}(x)$,take the inverse Shehu transform of each term of the equation (17), we obtain

$$\mathcal{V}(x) = \mathbb{S}^{-1}\left\{\frac{u^2}{s^2}\right\}$$

This implies

$$\mathcal{V}(x) = x$$

Substitute $V(x)$ back to find $u(x)$:

$$\sin u(x) = \mathcal{V}(x) = x$$

This implies

$$u(x) = sin^{-1}(x)$$

This is an exact solution.

VI. RESULTS & DISCUSSION

The nonlinear integral equations were solved using Rahmoh and Shehu transform–based semi-analytical approaches, and the results were evaluated through graphical comparisons with the corresponding exact solutions. The plots indicate that the computed solutions, shown by discrete points, align very closely with the exact solution curves across the entire interval. This close overlap demonstrates that the proposed techniques yield highly accurate approximations. Different types of solution behavior are observed, including gradual nonlinear variation, sharper changes near boundary regions, and periodic patterns. In each situation, the approximate results follow the exact solutions consistently without visible divergence. Even in regions where the solution changes rapidly, the methods maintain stability and preserve accuracy, which reflects their strong numerical performance.

The results also show that the proposed approaches are effective for handling various forms of nonlinearities. The approximation error is minimal, and the convergence toward the exact solution is achieved efficiently with relatively low computational effort. The agreement between analytical and numerical outcomes across all graphical representations confirms the reliability of these techniques. Overall, Rahmoh and Shehu transform-based methods provide simple, accurate, and efficient solutions for nonlinear integral equations, and they can be confidently applied to a wide range of problems in applied mathematics and related disciplines.

For graphical representations and numerical computations, the computational software MATLAB has been used.

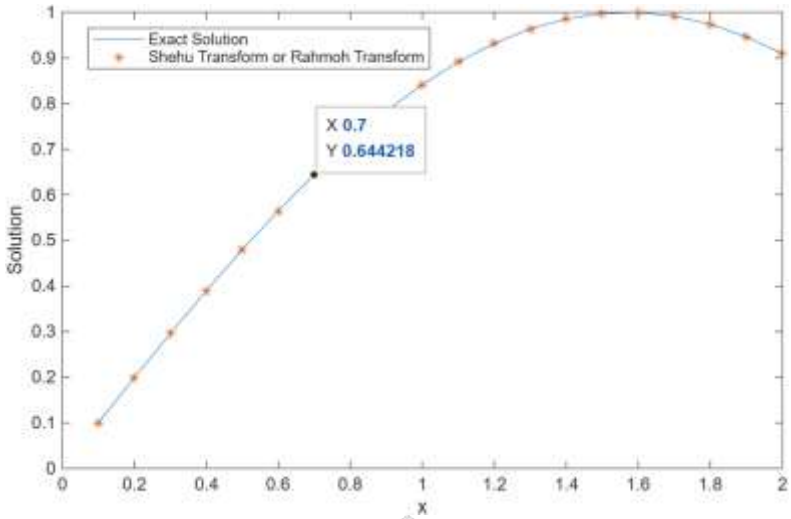

Figure 1: Comparison of exact solutions with solutions obtained by transform method

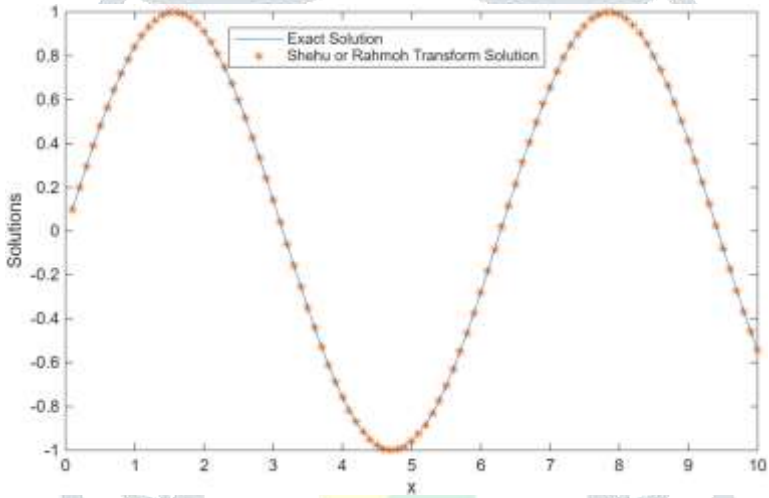

Figure 2: Comparison of exact solutions with solutions obtained by integral transform method

Figure 1 and Figure 2 represents the comparison of exact solution and solutions obtained by using Shehu transform or Rahmoh transform of Example 1 for different range of x.

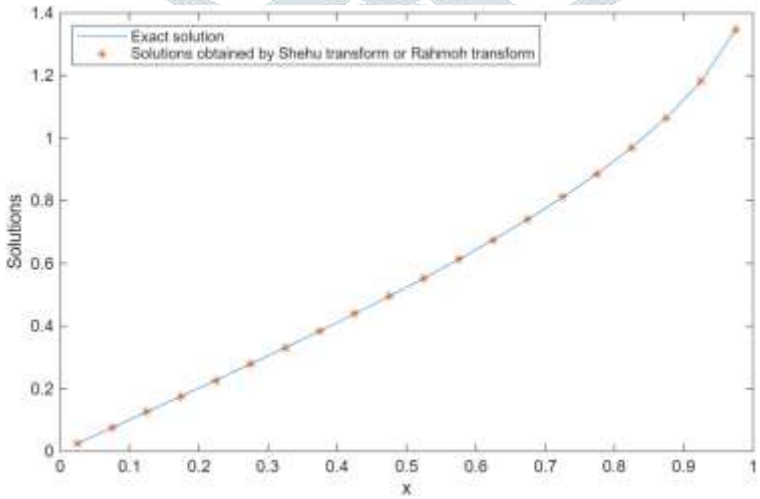

Figure 3: Comparison of exact solutions and solutions obtained by integral transform

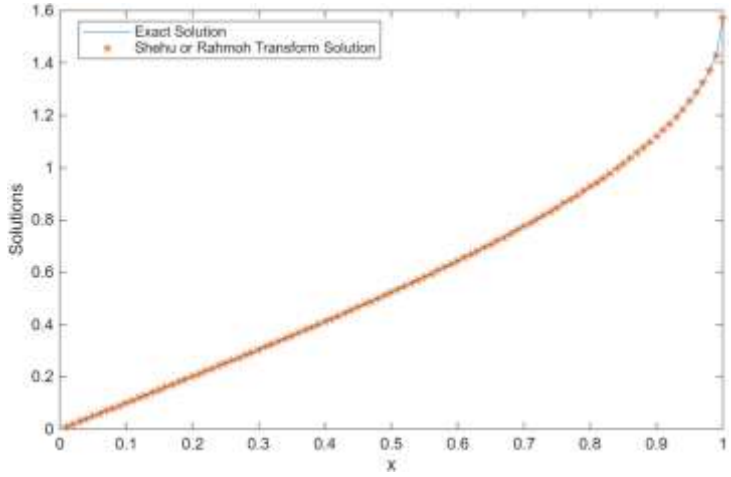

Figure 4: Comparison of exact solutions and solutions obtained by integral transform

Figure 3 and Figure 4 represents the comparison of exact solution and solutions obtained by using Shehu transform or Rahmoh transform of Example 2 for different range of x.

VII. CONCLUSION

This study presents two efficient semi-analytical methods based on Rahmoh and Shehu transforms for solving nonlinear integral equations. The proposed approaches simplify the solution process and provide accurate results with reduced computational effort. Hence, these transform-based techniques serve as useful and practical tools for handling nonlinear integral equations of the first kind, arising in several applications of sciences and engineering.

VIII. ACKNOWLEDGMENT

We are grateful to the anonymous reviewers for their valuable comments, which have significantly improved the manuscript.

REFERENCES

[1] S. Aggarwal, A. R. Gupta and S. D. Sharma, "A new application of Shehu transform for handling Volterra integral equations of first kind," *International Journal of Research in Advent Technology*, vol. 7, no. 4, pp. 190–198, 2019. DOI: 10.32622/ijrat.742019151

[2] S. Aggarwal, S. D. Sharma and A. R. Gupta, "Application of Shehu transform for handling growth and decay problems," *Global Journal of Engineering Science and Researches*, vol. 6, no. 4, pp. 190–198, 2019. DOI:10.5281/zenodo.2646649

[3] S. Maitama and W. Zhao, "New integral transform: Shehu transform, a generalization of Sumudu and Laplace transform for solving differential equations," *International Journal of Analysis and Applications*, vol. 17, no. 2, pp. 167–190, 2019. DOI: 10.28924/2291-8639-17-2019-167

[4] S. Aggarwal and A. R. Gupta, "Shehu transform for solving Abel's integral equation," *Journal of Emerging Technologies and Innovative Research*, vol. 6, no. 5, pp. 101–108, 2019.

[5] R. Belgacem, D. Baleanu and A. Bokhari, "Shehu transform and applications to Caputo-fractional differential equations," *International Journal of Analysis and Applications*, vol. 17, no. 6, pp. 917–927, 2019.DOI:10.28924/2291-8639-17-2019-917

[6] A. Issa and Y. Mensah, "Shehu transform: Extension to distributions and measures," *Journal of Nonlinear Modeling and Analysis*, vol. 2, no. 4, pp. 541–549, 2020.DOI: 10.12150/jnma.2020.541

[7] M. Andualem and A. Asfaw, "Application of Shehu transform to mechanics, Newton's law of cooling and electric circuit problems," *International Journal of Scientific Research and Engineering Development*, vol. 3, no. 4, pp. 372–378, 2020.

[8] A. N. Albukhattar, Z. D. Ridha and H. N. Kadhim, "Applications of a Shehu transform to the heat and transport equations," *International Journal of Psychosocial Rehabilitation*, vol. 24, no. 5, pp. 4251–4258, 2020.

[9] N. D. Dhange, "Shehu transform for solution of some integral equations," *Journal of Emerging Technologies and Innovative Research*, vol. 8, no. 9, pp. c747–c753, 2021.

[10] A. S. V. R. Kanth, K. Aruna, K. Raghavendar, H. Rezazadeh and M. Inc, "Numerical solutions of nonlinear time fractional Klein-Gordon equation via natural transform decomposition method and iterative Shehu transform method," *Journal of Ocean Engineering and Science*, vol. 7, no. 6, pp. 572–582, 2021. DOI: 10.1016/j.joes.2021.12.002

[11] A. N. Albukhuttar, B. S. Jubear and M. T. Neamah, "Solve the Laplace, Poisson and Telegraph equations using the Shehu transform," *Turkish Journal of Computer and Mathematics Education*, vol. 12, no. 10, pp. 1759–1768, 2021.

[12] T. G. Thange and A. R. Gade, "Fractional Shehu transform and its applications," *South East Asian Journal of Mathematics and Mathematical Sciences*, vol. 17, no. 2, pp. 1–14, 2021.

[13] M. Kapoor, N. A. Shah, S. Saleem and W. Weera, "An analytical approach for fractional hyperbolic telegraph equation using Shehu transform in one, two and three dimensions," *Mathematics*, vol. 10, no. 12, 1961, 2022. DOI: 10.3390/math10121961

[14] V. Govindan, S. Noeiaghdam, U. Fernandez-Gamiz, S. N. Sankeshwari, R. Arulprakasam and B. Z. Li, "Shehu integral transform and Hyers-Ulam stability of nth order linear differential equations," *Scientific African*, vol. 18, e01427, 2022.DOI: 10.1016/j.sciaf.2022.e01427

[15] R. Shah, A. S. Alshehry and W. Weera, "A semi-analytical method to investigate fractional-order gas dynamics equations by Shehu transform," *Symmetry*, vol. 14, no. 7, 1458, 2022. DOI:10.3390/sym14071458

[16] S. Alfaqeih, G. Bakicierler and E. Misirli, "Application of double Shehu transforms to Caputo fractional partial differential equations," *Punjab University Journal of Mathematics*, vol. 54, no. 1, pp. 1–13, 2022.
DOI: 10.52280/pujm.2021.540101

[17] B. M. Yisa and M. A. Baruwa, "Shehu transform Adomian decomposition method for the solution of linear and nonlinear integral and integro-differential equations," *Journal of the Nigerian Mathematical Society*, vol. 41, no. 2, pp. 107–120, 2022.

[18] T. G. Thange and A. M. Alure, "Generalized Shehu transform," *Journal of Mathematics and Computer Science*, vol. 12, Article 27, pp. 1–11, 2022.DOI:10.28919/jmcs/6922

[19] M. Al-Momani, A. Jaradat, B. Abughazaleh and A. Farah, "Solving partial differential equations via the double Sumudu-Shehu transform," *European Journal of Pure and Applied Mathematics*, vol. 18, no. 2, 5898, 2025. DOI: 10.29020/nybg.ejpam.v18i2.5898

[20] L. H. Al-Taee, A. A. M. Fawze and N. L. Housen, "Applications of Shehu transform in integral and integro-differential equations," *AIP Conference Proceedings*, vol. 3264, no. 1, Article 050115, 2025. DOI: 10.1063/5.0258942

[21] R. A. Farah and M. A. Hamad, "On the use of RAHMOH integral transform for solving differential equations," *International Journal of Physics and Mathematics*, vol. 6, no. 2, pp. 1–8, 2024.DOI:10.33545/26648636.2024.v6.i2a.83

[22] M. Hamad, M. Adam, O. Y. Olatunji and R. Farah, "A rigorous review of recent integral transforms for solving differential equations: Analysis, properties and applications," *Earthline Journal of Mathematical Sciences*, vol. 16, no. 3, pp. 373–391, 2026.DOI: 10.34198/ejms.16326.27.373391

[23] Danish-Ul-Islam and I. Singh, "Hybrid numerical methods for solving Volterra integral equation of the first kind," *International Journal of Advanced Research in Engineering and Technology (IJARET)*, vol. 11, no. 10, pp. 694–705, 2020.

[24] I. Singh and U. Kumari, "Elzaki transform homotopy perturbation method for solving two-dimensional time-fractional Rosenau-Hyman equation," *MATEMATIKA, MJIM*, vol. 39, no. 2, pp. 159–171, 2023.

[25] I. Singh and U. Kumari, "Coupling of Elzaki transform and HPM for solving 2D wave propagation equation," *AIP Conference Proceedings*, vol. 3185, no. 1, Art. no. 020060, 2025.

[26] I. Singh and U. Kumari, "Solving 2D and 3D telegraph equations with Elzaki transform and homotopy perturbation method," *Journal of Mechanics of Continua and Mathematical Sciences*, Special Issue, no. 11, pp. 18–29, 2024.

[27] I. Singh, N. H. Ansari and G. Singh, "Solving PDEs arising in the formation of liquid drop pattern using Sumudu transform based technique," *Partial Differential Equations in Applied Mathematics*, vol. 8, Art. no. 100578, 2023.