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# COEFFICIENT ESTIMATES FOR A SUBCLASS OF UNIVALENT FUNCTIONS IN THE UNIT DISK

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## Abstract

We consider normalized analytic functions f in the open unit disk whose derivative has positive real part. This condition guarantees univalence in the disk and permits coefficient estimates through the classical Carathéodory coefficient inequality. If f(z) = z + Σ_{n≥2} a_n z^n and Re f′(z) > 0 for |z| < 1, then |a_n| ≤ 2/n for every n ≥ 2. We give a direct proof, discuss the role of the hypothesis, and illustrate the estimate with an example. The note is intended as a concise expository mathematical manuscript; the result is a consequence of classical theory and is not claimed as a new theorem.

**Keywords**: univalent functions, analytic functions, coefficient estimates, positive real part, Carathéodory inequality.

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## 1. Introductions

A holomorphic function is called univalent in a domain if it is one-to-one there. Univalent functions are central in geometric function theory because they describe conformal mappings and connect analytic properties with the geometry of image domains. Let $D = \{z \in C : |z| < 1\}$ be the open unit disk. A standard normalized analytic function has the expansion

$$f(z) = z + a_2z^2 + a_3z^3 + \cdots, \quad z \in D.$$

The class S consists of all such functions that are univalent in D. Exact coefficient questions for S are deep; the Bieberbach theorem states that $|a_n| \leq n$ for every $n \geq 2$, with equality attained by rotations of the Koebe function. A simpler sufficient condition for univalence is Re f′(z) > 0 throughout a convex domain. In this note, we use that condition to obtain an elementary coefficient bound for a subclass of S.

## 2. Preliminaries

Definition 2.1. Let P denote the Carathéodory class of analytic functions p in D satisfying p(0) = 1 and Re p(z) > 0 for $z \in D$.

Lemma 2.2 (Carathéodory coefficient inequality). If $p(z) = 1 + \Sigma_{k\geq1} c_kz^k$ belongs to P, then $|c_k| \leq 2$ for every $k \geq 1$.

Lemma 2.3 (Univalence criterion). If f is analytic in a convex domain Ω and Re f′(z) > 0 for every $z \in \Omega$, then f is univalent in Ω.

For the unit disk, the last statement can be seen directly. For distinct $z_1, z_2 \in D$, convexity ensures the segment between them lies in D, and

$f(z_2) - f(z_1) = (z_2 - z_1) \int_0^1 f'(z_1 + t(z_2 - z_1))\, dt.$

The integral has positive real part, so it cannot be zero. Hence $f(z_2) \neq f(z_1)$.

## 3. Main result

Theorem 3.1. Suppose f is analytic in D and has the normalized expansion $f(z) = z + \Sigma_{n\geq 2} a_n z^n$. If $\mathrm{Re}\, f'(z) > 0$ for all $z \in D$, then f is univalent in D and

**$|a_n| \leq 2/n, \quad n \geq 2.$**

Proof. Since $\mathrm{Re}\, f'(z) > 0$ and $f'(0) = 1$, the function $p(z) = f'(z)$ belongs to the Carathéodory class P. Differentiating the Taylor expansion gives

$p(z) = f'(z) = 1 + \Sigma_{n\geq 2} n\, a_n\, z^{n-1}.$

The coefficient of $z^{n-1}$ in p is $c_{n-1} = n\, a_n$. By Lemma 2.2, $|c_{n-1}| \leq 2$. Therefore $n|a_n| \leq 2$, which yields $|a_n| \leq 2/n$ for every $n \geq 2$. Lemma 2.3 shows that f is univalent in D. This completes the proof.

## 4. Example

Consider $f(z) = z/(1 - z) = z + z^2 + z^3 + \cdots$ for $|z| < 1$. Its derivative is $f'(z) = 1/(1 - z)^2$. This particular function does not satisfy $\mathrm{Re}\, f'(z) > 0$ throughout the disk, so it is not an example for Theorem 3.1. Instead, take

$f(z) = 2z - \log(1 + z),$

with the logarithm defined by its analytic branch in D. Then $f(0) = 0$ and $f'(z) = 2 - 1/(1 + z) = (1 + 2z)/(1 + z)$; positivity of the real part is not immediate throughout D, so this example is also not used to assert sharpness.

A clean family satisfying the hypothesis is $f(z) = z + \lambda z^2$ for real $\lambda$ with $|\lambda| < 1/4$. Indeed, $f'(z) = 1 + 2\lambda z$ and $\mathrm{Re}\, f'(z) \geq 1 - 2|\lambda| > 1/2$ on D. The theorem gives $|a_2| = |\lambda| \leq 2/2 = 1$, which holds, although it is not sharp for this smaller family. This example illustrates how the general coefficient estimate can be applied without claiming equality.

## 5. Discussion and conclusion

The positive-real-part condition on the derivative is a convenient sufficient condition for univalence. Its advantage is that it turns a geometric one-to-one property into a coefficient problem for a function in the Carathéodory class. The resulting estimate $|a_n| \leq 2/n$ is straightforward and useful for this subclass, but it should not be confused with the sharp general estimate $|a_n| \leq n$ for the full class S. Further work could investigate sharper bounds under additional hypotheses, such as restrictions on the second coefficient or on the real part of a suitable differential expression. Any such extension would require a separate proof and a careful comparison with existing literature.

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## Author note before submission

This manuscript presents a classical consequence of the Carathéodory coefficient inequality. It is suitable as a study draft or starting point, not as a claim of original research by itself. Before journal submission, verify all bibliographic details, add the correct institutional affiliation and email, and develop a genuinely new result supported by a literature review. Do not describe the theorem as novel unless novelty has been established.